CHAPTER

13

Radiation

13.1 Introduction

So far, we considered heat transfer by conduction and convection. In these modes of heat transfer, there was always a medium present for heat transfer to occur. However, radiation mode of heat transfer is radically different in the sense that there is no need for a medium to be present for heat transfer to occur. Just as conduction and convection heat transfers occur when there is a temperature gradient, net radiation heat transfer also occurs from a higher temperature level to a lower temperature level. There are two theories concerning the radiation heat transfer: one, classical electromagnetic wave theory of Maxwell, according to which energy is transferred during radiation by electromagnetic waves, which travel as rays and follow the laws of optics; second, the 'Quantum theory' of physics, according to which energy is radiated in the form of successive, discrete 'quanta' of energy, called 'photons'. Both the theories are useful to explain the radiation phenomenon and properties.

Radiation heat transfer is proportional to the fourth power of absolute temperature of the radiating surface. Therefore, radiation becomes the predominant mode of heat transfer when the temperature of the body is high. With this in mind, we can cite a few important applications of radiation heat transfer:

(i) industrial heating, such as in furnaces
(ii) industrial air-conditioning, where the effect of solar radiation has to be considered in calculating the heat loads
(iii) jet engine or gas turbine combustors
(iv) industrial drying
(v) energy conversion with fossil fuel combustion, etc.

Following are some of the features of radiation:

(a) The electromagnetic magnetic waves are of all wavelengths, travelling at the velocity of light, i.e. $c = 3 \times 10^{10}$ cm/s
(b) Frequency (f) and wavelength (λ) are connected by the relation: $c = \lambda.f$, which means that higher the frequency, lower the wavelength
(c) Smaller the wavelength, more powerful is the radiation, and also more damaging, e.g. X-rays and Gamma rays.

A sketch of the electromagnetic spectrum is shown in Fig. 13.1. Different parts of the electromagnetic spectrum have wavelengths (λ) as shown in Table 13.1.

In this chapter, we are interested in radiations, which on absorption, result in production of heat, i.e. 'thermal radiation'. It may be observed that thermal radiation falls in the wavelength range of 0.1 to 100 microns (Unit of wavelength is 1 micron = 10^{-6} m, and 1 Angstrom = 10^{-10} m), i.e. thermal radiation includes entire visible (i.e. λ = 0.4 to 0.8 microns) and infra-red and part of ultra-violet range. As a matter of interest, it may be stated that most of the radiation from the sun (temperature: 5600°C approximately) is in the lower end of 0.1 to 0.4 microns and, for comparison, radiation from an incandescent lamp is in the range of 1 to 10 microns.

While most of the solids and liquids emit radiation in a continuous spectrum, gases and vapours radiate only in certain wavelength bands; therefore, they are known as 'selective emitters'.

TABLE 13.1 Wavelengths of different types of radiation

Kind of radiation	Wavelength, λ
Cosmic rays	up to 4×10^{-7} μm
Gamma rays	4×10^{-7} to 1.4×10^{-4} μm
X-rays	1.0×10^{-5} to 2×10^{-2} μm
Ultra-violet	0.01 to 0.39 μm
Visible radiation	0.39 to 0.78 μm
Thermal radiation	0.1 to 100 μm
Infra-red	0.78 to 1000 μm
Microwave	0.8 to 1000 mm
Radio waves	Beyond 1 m

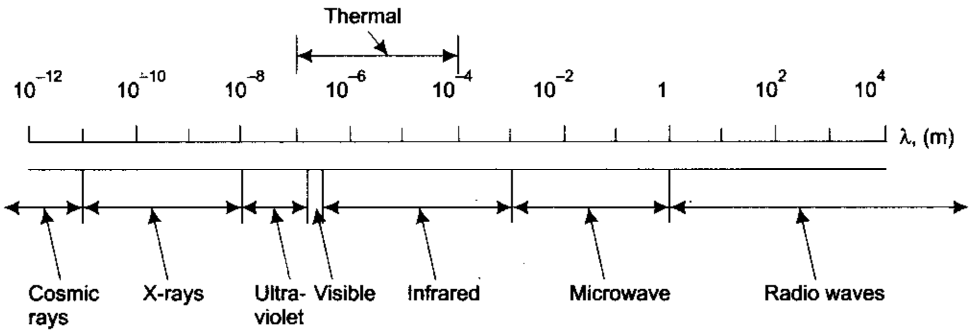


FIGURE 13.1 Electromagnetic spectrum

13.2 Properties and Definitions

Often, we use the term **'spectral'**; it means, dependence on wavelength.
And, value of a quantity at a given wave length is called **'monochromatic value'**.

Absorptivity, Reflectivity and Transmissivity:

In general, when radiant energy (Q_o) is incident on a surface, part of it may be absorbed (Q_a), part may be reflected (Q_r) and part may be transmitted (Q_t) through the body. Then, obviously,

$$Q_a + Q_r + Q_t = Q_o$$

i.e. $$\frac{Q_a}{Q_o} + \frac{Q_r}{Q_o} + \frac{Q_t}{Q_o} = 1$$

i.e. $$\alpha + \rho + \tau = 1 \quad ...(13.1)$$

where, α = absorptivity = fraction of incident radiation absorbed
ρ = reflectivity = fraction of incident radiation reflected
τ = fraction of incident radiation transmitted.

Most of the solids and liquids are 'opaque', i.e. they do not transmit radiation, and $\tau = 0$; so, for most solids and liquids: $\alpha + \rho = 1$.

Gases reflect very little; so, for gases: $\alpha + \tau = 1$.

If $\tau = 1$, entire radiation passes through the body; such a body is **'transparent'** or **'diathermaneous'**.

If $\alpha = 1$, the body absorbs all the incident radiation and such a body is called a **'black body'**.

If $\rho = 1$, all the incident radiation is reflected, and it is a perfectly **'white body'**.

In reality, there are no 'perfectly' black, white or transparent bodies.

However, some bodies are transparent to only waves of certain wavelength; for example, rock salt is transparent to heat rays, but non-transparent to ultra-violet rays. And, window glass is transparent to visible light, but almost non-transparent to ultra-violet and infra-red rays. Therefore, a space covered with glass (or plastic) enclosure, allows solar radiation to pass through it and the objects inside the enclosure get heated up; the heated objects radiate, but this radiation is in the higher wavelength range (infra-red) to which glass or plastic is opaque. So, the heat gets 'trapped' inside the enclosure and the temperature inside the enclosure rises above that of ambient. This is known as **'Greenhouse effect'** and is used to keep the plants warm in cold weather. Another example of Greenhouse effect is manifested in heating up the interior of a car to a temperature much above the ambient temperature when the car is parked in hot sun, with all its windows closed.

Absorption and reflection of heat rays depend rather on the state of the surface than on the colour of the surface. For example, snow has an absorptivity of 0.985 and is nearly 'black' for thermal radiation!

Absorptivity of a surface can be increased by applying coatings of dark paints; usually, lamp black is used for this purpose.

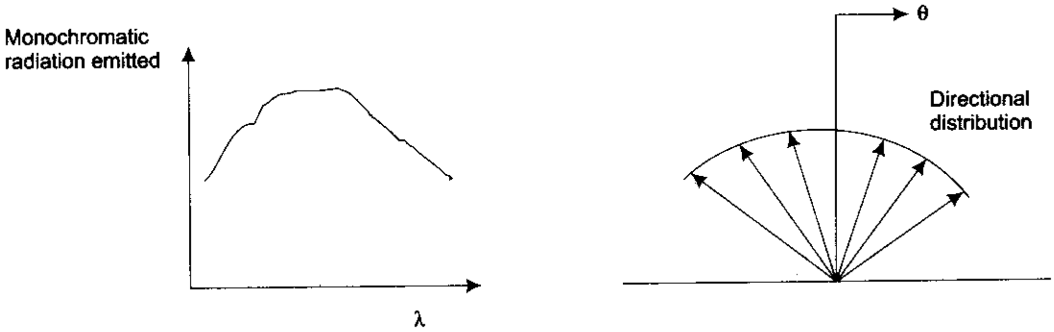


FIGURE 13.2 Spectral and spatial energy distribution

Spectral and Spatial energy distribution:
Distribution of radiant energy is non-uniform with respect to *both* wavelength and direction, as shown in Fig. 13.2.

Perfect black body A perfect black body does not exist in nature; however, a perfect black body can be approximated in the laboratory by having a sphere coated black on the inside; then, if there is a small hole on the wall of the sphere, the radiation Q entering the hole goes through multiple reflections and after 'n' reflections, $\rho^n.Q$ is the emergent energy flux. Obviously, the emerging flux tends to be zero when 'n' tends to infinity, i.e. the pin hole in the sphere simulates a black body. This is known as 'Hohlraum'. See Fig. 13.3.

Note the following points in connection with a black body:

(i) A black body absorbs all the incident radiation, of all wavelengths and from all directions
(ii) For a given temperature and wavelength, energy emitted by a black body is the maximum as compared to any other body
(iii) Black body is a 'diffuse emitter', i.e. the radiation emitted by a black body is independent of direction
(iv) A black body does not reflect or transmit any of incident radiation

Reflection Reflection may be 'specular' (or mirror-like) or 'diffuse'. See Fig. 13.4

In specular reflection, the angle of incidence is equal to the angle made by the reflected ray with the normal to the surface. In case of diffuse reflection, magnitude of reflected energy in a given direction is proportional to the cosine of the angle of that direction to the normal. 'Roughness' of the surface determines if the reflection is specular or diffuse: if the 'height' of corrugations on the surface is much smaller than the wavelength of incident radiation, the surface behaviour is specular; otherwise, it is diffuse.

Emissive power (*E*) The 'total (or hemispherical) emissive power' is the total thermal energy radiated by a surface per unit time and per unit area, over all the wavelengths and in all directions. Note, in particular, that only the original, emitted energy is to be considered and the reflected energy is *not* to be included. Total emissive power depends on the temperature, material and the surface condition.

Solid angle 'Solid angle' is defined as a region of a sphere, which is enclosed by a conical surface with the vertex of the cone at the centre of the sphere. See Fig. 13.5.

If there is a source of radiation of a small area at the centre of the sphere O, then the radiation passes through the area A_n on the surface of the sphere and we say that the area A_n subtends a solid angle ω when viewed from the centre of the sphere. Note that with this definition, A_n is always normal to the radius of the sphere. Mathematically, solid angle is expressed as:

$$\omega = \frac{A_n}{r^2} \qquad \text{(steradians (sr)...(13.2))}$$

However, in a practical case, the surface may not be part of a sphere; but, if a plane area A intercepts the line of propagation of radiation such that the normal to the surface makes an angle θ with the line of propagation, then we project the incident area normal to the line of propagation, such that, the solid angle is now defined as:

$$\omega = \frac{A \cdot \cos(\theta)}{r^2}\ sr \qquad \text{...(13.3)}$$

Note that, $A.\cos(\theta) = A_n$ is the projected area of the incident surface, normal to the line of propagation.

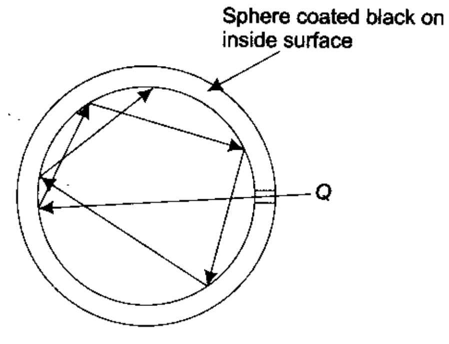


FIGURE 13.3 Simulation of a black body in laboratory—'Hohlraum'

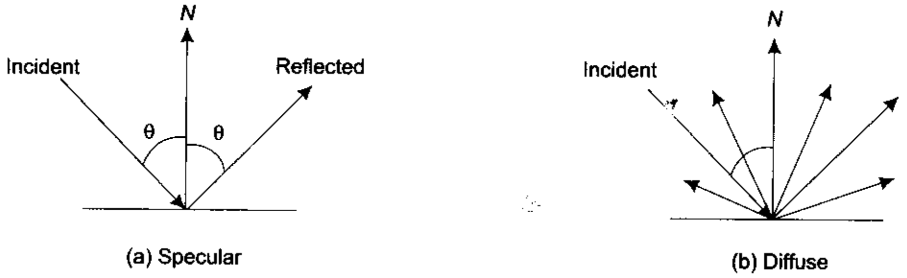


FIGURE 13.4 Specular and diffuse reflection

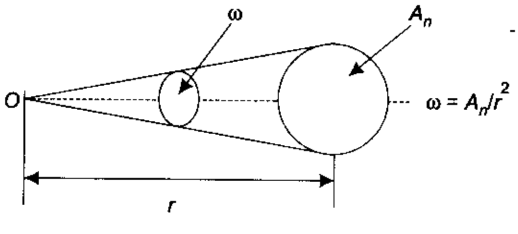


FIGURE 13.5 Definition of solid angle

Intensity of radiation (I_b) Intensity of radiation for a black body, I_b is defined as the energy radiated per unit time per unit solid angle per unit area of the emitting surface projected normal to the line of view of the receiver from the radiating surface.

Mathematically, this is expressed as:

$$I_b = \frac{dQ_b}{(dA \cdot \cos(\theta)) \cdot d\omega}\ \text{W/(m}^2\text{sr)} \qquad \text{...(13.4)}$$

i.e.

$$I_b = \frac{dE_b}{\cos(\theta) \cdot d\omega}\ \text{W/(m}^2\text{sr)} \qquad \text{...(13.5)}$$

Note that Emissive power E_b of a black body refers to unit *surface area* whereas Intensity I_b of a black surface refers to unit *projected area*.

$I_{b\lambda}$ is the intensity of black body radiation for radiation of a given wavelength λ. And, I_b is the summation over all the wavelengths, i.e.

$$I_b = \int_0^{\infty} I_{b\lambda}\, d\lambda\ \text{W/(m}^2\text{sr)} \qquad \text{...(13.6)}$$

Consider a small, black surface dA emitting radiation all over a hemisphere above it. See Fig.13.6. Let a radiation collector be located on the hemispherical surface at a zenith angle θ to the normal to the surface and azimuth angle φ; further, let the collector subtend a solid angle $d\omega$ when viewed from a point on the emitter. Then, it will be observed that maximum amount of radiation is measured when the collector is vertically above the emitter, normal to the emitter. In any direction θ from the normal, rate of energy radiated is given by **Lambert's cosine law:** "A diffuse surface radiates energy such that the rate of energy radiated in a direction θ from the normal to the surface is proportional to the cosine of the angle θ", i.e.

$$Q_\theta = Q_n \cdot \cos(\theta)$$

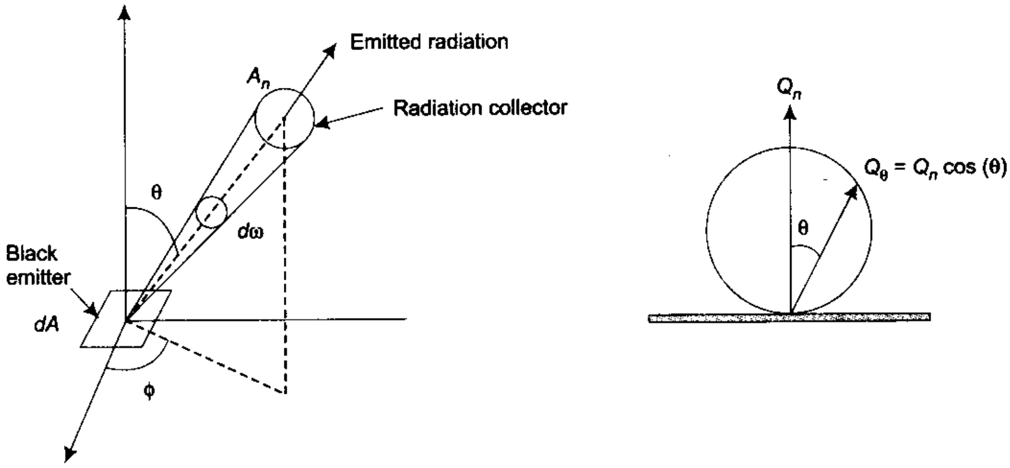


FIGURE 13.6 Lambert's cosine law

In general, for real surfaces, intensity does not vary with ϕ, but depends on θ, however, with the intensity of radiation as defined above, i.e. on basis of unit projected area, it can be shown that for a black surface, the intensity is the same in all directions. Such a surface is known as 'diffuse surface'.

For a diffuse, black surface, radiation intensity is independent of direction and such surfaces are also known as 'Lambertonian surfaces'.

Intensity can be thought of as brightness; looking down vertically along the normal, a viewer sees all of the black surface dA at a particular level of brightness; and looking down along a line that makes an angle θ with the normal, the viewer will see only the projected area $dA.\cos(\theta)$, but at the same level of brightness.

Many real bodies, which are not diffuse, do not obey Lambert's law and their radiation intensity changes with the direction θ; for example, for polished metals, the 'brightness' is a maximum not in the direction normal to the surface, but at 60 to 80 deg. from the normal, and with further increase in θ, the brightness drops abruptly to zero. But for materials like corundum and copper oxide, the intensity (or brightness) is greater along the normal than that in other directions.

13.3 Laws of Black Body Radiation

13.3.1 Planck's Law for Spectral Distribution

Radiation energy emitted by a black surface depends on the wavelength, temperature of the surface and the surface characteristics.

Planck's distribution law relates to the **spectral black body emissive power, $E_{b\lambda}$** defined as 'the amount of radiation energy emitted by a black body at an absolute temperature T per unit time, per unit surface area, per unit wavelength about the wavelength λ'. Units of $E_{b\lambda}$ are: W/(m$^2\mu$m). The first subscript 'b' indicates black body and the second subscript 'λ' stands for given wavelength, or monochromatic. Planck derived his equation for $E_{b\lambda}$ in 1901 in conjunction with his 'quantum theory'.

Planck's distribution law is expressed as:

$$E_{b\lambda}(\lambda) = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} \quad \text{W/(m}^2\mu\text{m)} \qquad \text{...(13.7)}$$

where, $C_1 = 3.742 \times 10^8$ Wμm^4/m^2

and, $C_2 = 1.4387 \times 10^4$ μmK.

Plots of $E_{b\lambda}$ vs. λ for a few different temperatures are shown in Fig. 13.7.

To plot the Planck's distribution for a black body, using Mathcad, first, define $E_{b\lambda}$ as a function of T and λ as follows:

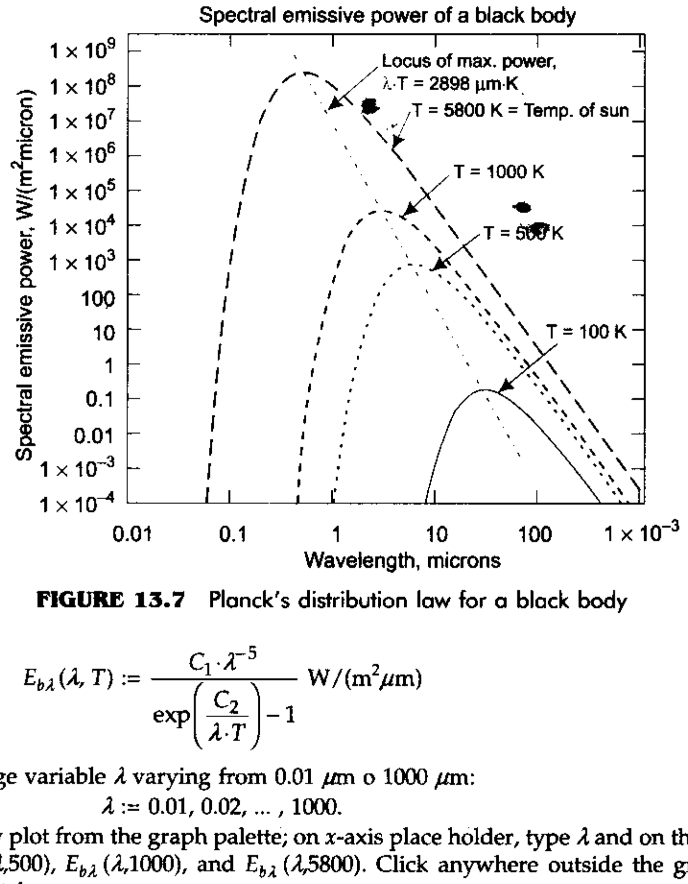


FIGURE 13.7 Planck's distribution law for a black body

$$E_{b\lambda}(\lambda, T) := \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} \quad W/(m^2 \mu m) \qquad ...(13.7)$$

Then, define a range variable λ varying from 0.01 μm o 1000 μm:

$$\lambda := 0.01, 0.02, \ldots, 1000.$$

Now, select the x–y plot from the graph palette; on x-axis place holder, type λ and on the y-axis place holder, fill in $E_{b\lambda}(\lambda,100)$, $E_{b\lambda}(\lambda,500)$, $E_{b\lambda}(\lambda,1000)$, and $E_{b\lambda}(\lambda,5800)$. Click anywhere outside the graph region, and the curves appear immediately.

This is an important graph that tells us quite a lot about the characteristics of black body radiation:

(i) At a given absolute temperature T, a black body emits radiation over all wavelengths, ranging from 0 to ∞.

(ii) Spectral emissive power curve varies continuously with wavelength.

(iii) At a given wavelength, as temperature increases, emissive power also increases.

(iv) At a given temperature, emissive power curve goes through a peak, and a major portion of the energy radiated is concentrated around this peak wavelength λ_{max}.

(v) A significant part of the energy radiated by sun (considered as a black body at a temperature of 5800 K) is in the visible region (λ = 0.4 to 0.7 microns), whereas a major part of the energy radiated by earth at 300 K falls in the infra-red region.

(vi) As temperature increases, the peak of the curve shifts to the left, i.e. towards the shorter wavelengths.

(vii) Area under the curve between λ and $(\lambda + d\lambda) = E_{b\lambda}.d\lambda$ = radiant energy flux leaving the surface within the range of wavelength λ to $(\lambda + d\lambda)$. Integrating over the entire range of wavelengths,

$$E_b = \int_0^{\infty} (E_\lambda)\, d\lambda = \sigma \cdot T^4 \qquad ...(13.8)$$

E_b is the total emissive power (also known as 'radiant energy flux density') per unit area radiated from a black body, and σ is the Stefan–Boltzmann constant = $5.67 \times 10^{-8} W/(m^2K^4)$.

Corollaries of Planck's law:

(a) For shorter wavelengths, $(C_2/\lambda.T)$ becomes very large, and $\exp(C_2/\lambda.T) >> 1$. Then, Planck's formula (Eq. 13.7) reduces to:

$$E_{b\lambda} = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right)} \quad ...(13.9)$$

This equation is known as **'Wein's law'** and is accurate within 1 % for $\lambda.T < 3000$ μmK.

(b) For longer wavelengths, the factor $(C_2/\lambda.T)$ becomes very small, and $\exp(C_2/\lambda.T)$ can be expanded in a series as follows:

$$\exp\left(\frac{C_2}{\lambda \cdot T}\right) = 1 + \frac{C_2}{\lambda \cdot T} + \frac{1}{2!}\cdot\left(\frac{C_2}{\lambda \cdot T}\right)^2 + ...$$

i.e.

$$\exp\left(\frac{C_2}{\lambda \cdot T}\right) = 1 + \frac{C_2}{\lambda \cdot T} \quad \textit{(approximate)}$$

and, Planck's law becomes:

$$E_{b\lambda} = \frac{C_1 \cdot \lambda^{-5}}{1 + \frac{C_2}{\lambda \cdot T} - 1} = \frac{C_1 \cdot T}{C_2 \cdot \lambda^4} \quad ...(13.10)$$

This is known as **'Rayleigh–Jean's law'** and is accurate within 1 % for ll. $\lambda T > 8 \times 10^5$ μmK. This law is useful in analysing long wave radiations such as radio waves.

13.3.2 Wein's Displacement Law

It is clear from Fig. 13.7 that the spectral distribution of emissive power of a black body at a given absolute temperature goes through a maximum. To find out the value of λ_{max}, the wavelength at which this maximum occurs, differentiate Planck's equation w.r.t. λ and equate to zero. We get:

$$E_{b\lambda} = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} \quad \textit{(from Eq. 13.7)}$$

i.e.

$$\frac{dE_{b\lambda}}{d\lambda} = \frac{\left(\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1\right)\cdot C_1 \cdot (-5)\cdot \lambda^{-6} - C_1 \cdot \lambda^{-5} \cdot \left(\exp\left(\frac{C_2}{\lambda \cdot T}\right)\right)\cdot\left(\frac{C_2}{T}\right)\cdot(-1)\cdot \lambda^{-2}}{\left(\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1\right)^2} = 0$$

Simplifying,

$$5 \cdot \left(1 - \exp\left(\frac{-C_2}{\lambda \cdot T}\right)\right) - \frac{C_2}{\lambda \cdot T} = 0$$

Solving this transendental Eq. for $C_2/\lambda.T$ by trial and error, we get:

$$\frac{C_2}{\lambda \cdot T} = 4.965$$

Therefore,

$$\lambda_{max} \cdot T = \frac{C_2}{4.965} = \frac{1.4387 \times 10^4}{4.965}$$

i.e.

$$\lambda_{max} \cdot T = 2898 \ \mu\text{mK} \quad ...(13.11)$$

i.e. λ_{max} is inversely proportional to the absolute temperature T, and the maximum spectral intensity shifts towards shorter wavelengths as the absolute temperature is increased.

Wein's displacement law is stated as: "product of absolute temperature and wavelength at which emissive power of a black body is a maximum, is constant".

Value of maximum monochromatic emissive power of a black body at a given temperature is obtained by substituting this value of $\lambda_{max}T$ (= 2898 μmK) in Planck's equation, i.e.

$$E_{b\lambda_{max}} = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} = \frac{3.742 \times 10^{-16} \cdot \left(\frac{0.002898}{T}\right)^{-5}}{\exp\left(\frac{1.4387 \times 10^4}{2898}\right) - 1}$$

i.e.

$$E_{b\lambda_{max}} = 1.287 \times 10^{-5} \cdot T^5 \text{ W/m}^3. \qquad ...(13.12)$$

This is an important equation which tells us that the maximum monochromatic emissive power of a black body varies as the fifth power of the absolute temperature of the body.

In practice, this law is applied to predict very high temperatures simply by measuring the wavelength of radiation emitted.

Dividing monochromatic emissive power of a black body, $E_{b\lambda}$, by its maximum emissive power at the same temperature, $E_{b\lambda_{max}}$, we get the dimensionless ratio:

$$\frac{E_{b\lambda}(T)}{E_{b\lambda\max}(T)} = \frac{\dfrac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1}}{\dfrac{C_1 \cdot \lambda_{max}^{-5}}{\exp\left(\frac{C_2}{\lambda_{max} \cdot T}\right) - 1}}$$

i.e.

$$\frac{E_{b\lambda}(T)}{E_{b\lambda\max}(T)} = \left(\frac{2898 \times 10^{-6}}{\lambda \cdot T}\right)^5 \cdot \left(\frac{\exp(4.965) - 1}{\exp\left(\frac{0.01439}{\lambda \cdot T}\right) - 1}\right)$$

(where, λ is in microns, and T in Kelvin...(13.12a))

Note that RHS of Eq. 13.12a is a function of λT only. Therefore, to determine the monochromatic emissive power, $E_{b\lambda}$, of a black body at any given temperature T and wavelength λ, first find out ($E_{b\lambda}/E_{b\lambda max}$) from Eq. 13.12a, then evaluate $E_{b\lambda max}$ from Eq. 13.12, and then multiply them together.

13.3.3 Stefan–Boltzmann Law

Monochromatic emissive power of a black body is obtained from the Planck's law. Then, the total emissive power of a black body over the entire wavelength spectrum is obtained by integrating $E_{b\lambda}$. Total emissive power (or, hemispherical total emissive power) is denoted by E_b, and is given as:

$$E_b = \int_0^{\infty} E_{b\lambda}\, d\lambda = \int_0^{\infty} \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} d\lambda$$

Performing the integration, we get:

$$E_b = \sigma \cdot T^4 \text{ W/m}^2 \qquad ...(13.13)$$

where, $\sigma = 5.67 \times 10^{-8}$ W/(m^{2}K^4)

σ is known as **'Stefan–Boltzmann constant'.**

Eq. 13.13 is the governing rate equation for radiation from a black body. Its significance lies in the fact that just with a knowledge of the absolute temperature of a surface, one can calculate the total amount of energy radiated in all directions over the entire wavelength range.

Net radiant energy exchange between two black bodies at temperatures T_1 and T_2 is, therefore, given by:

$$Q_{net} = \sigma \cdot (T_1^4 - T_2^4) \text{ W/m}^2. \quad ...(13.14)$$

13.3.4 Radiation from a Wave Band

Often, it is required to know the amount of radiation emitted in a given wave band, i.e. in a wavelength interval between λ_1 and λ_2. This is expressed as a fraction of the total emissive power and is written as $F_{\lambda 1-\lambda 2}$. Then, we can write:

$$F_{\lambda 1\_\lambda 2} = \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{\int_0^{\infty} E_{b\lambda}\, d\lambda} = \frac{1}{\sigma \cdot T^4} \cdot \int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda$$

i.e.

$$F_{\lambda 1\_\lambda 2} = \frac{1}{\sigma \cdot T^4} \cdot \left(\int_0^{\lambda_2} E_{b\lambda}\, d\lambda - \int_0^{\lambda_1} E_{b\lambda}\, d\lambda \right)$$

i.e.

$$F_{\lambda 1\_\lambda 2} = F_{0\_\lambda 2} - F_{0\_\lambda 1}. \quad ...(13.15)$$

Above formula is not very convenient to use, since $E_{b\lambda}$ depends on absolute temperature T, and it is not practicable to tabulate $F_{0-\lambda}$ for each T. This difficulty is overcome by expressing $F_{0-\lambda}$ as follows:

$$F_{0\_\lambda} = \frac{\int_0^{\lambda} E_{b\lambda}\, d\lambda}{\sigma \cdot T^4} = \frac{\int_0^{\lambda \cdot T} E_{b\lambda}\, d(\lambda \cdot T)}{\sigma \cdot T^5}$$

i.e.

$$F_{0\_\lambda} = f(\lambda \cdot T) \quad ...(13.16)$$

i.e. now, F_{0_λ} is expressed as a function of the product of wavelength and absolute temperature (= $\lambda . T$) only.

Values of F_{0_λ} vs. $\lambda . T$ are tabulated in Table 13.2 and plotted in Fig. 13.8.

Note that the units of product $\lambda . T$ is (micronKelvin).

Therefore,

$$F_{\lambda 1\_\lambda 2} = F_{\lambda 1 \cdot T\_\lambda 2 \cdot T} = \frac{1}{\sigma} \cdot \left[\int_0^{\lambda_2 \cdot T} \frac{E_{b\lambda}}{T^5} d(\lambda \cdot T) - \int_0^{\lambda_1 \cdot T} \frac{E_{b\lambda}}{T^5} d(\lambda \cdot T) \right]$$

i.e.

$$F_{\lambda 1\_\lambda 2} = F_{0\_\lambda 2 \cdot T} - F_{0\_\lambda 1 \cdot T}. \quad ...(13.17)$$

13.3.5 Relation between Radiation Intensity and Emissive Power

Consider a differential black emitter dA_1 radiating into a hemisphere of radius r, with the centre of the hemisphere located at dA_1. To get a relation between the intensity of radiation and the emissive power, we first

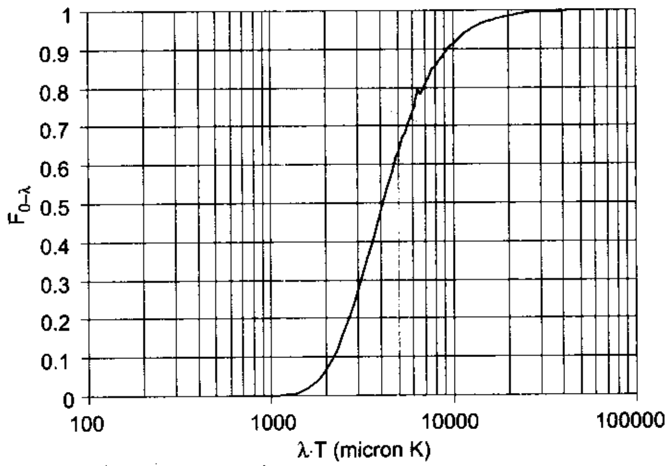


FIGURE 13.8 Fraction of black body radiation in the range (0–$\lambda \cdot T$)

TABLE 13.2 Radiation functions

λT (μmK)	$F_{0-\lambda}$	λT (μmK)	$F_{0-\lambda}$
400	0	7000	0.8081
600	0	7200	0.8192
800	0.000016	7400	0.8295
1000	0.00032	7600	0.848
1200	0.00213	7800	0.848
1400	0.0078	8000	0.8563
1600	0.0197	8500	0.8746
1800	0.0393	9000	0.89
2000	0.0667	9500	0.9031
2200	0.1009	10000	0.9142
2400	0.1403	10500	0.9237
2600	0.1831	11000	0.9319
2800	0.2279	11500	0.9399
3000	0.2732	12000	0.9451
3200	0.3181	12500	0.9505
3400	0.3617	13000	0.9551
3600	0.4036	13500	0.9592
3800	0.4434	14000	0.9628
4000	0.4809	14500	0.9661
4200	0.516	15000	0.9689
4400	0.5488	16000	0.9738
4600	0.5793	17000	0.9776
4800	0.6075	18000	0.9808
5000	0.6337	19000	0.9834
5200	0.659	20000	0.9855
5400	0.6804	25000	0.9922
5600	0.701	30000	0.9953
5800	0.7201	35000	0.9969
6000	0.7378	40000	0.9979
6200	0.7541	45000	0.9985
6400	0.7962	50000	0.9989
6600	0.7832	75000	0.9997
6800	0.7961	100000	0.9999

calculate the rate of energy falling on a differential area dA_2 on the surface of the hemisphere using the definition of intensity, then calculate the rate of energy falling on the whole of the hemisphere by integrating, and then equate this amount to the rate of radiant energy issuing from the black surface dA_1.

Let the rate of radiant energy falling on dA_2 be dQ. Solid angle subtended by dA_2 at the centre of the sphere, $d\omega = dA_2/r^2$. Projected area of dA_1 on a plane perpendicular to the line joining dA_1 and $dA_2 = dA_1.\cos(\theta)$. Then, by definition, intensity of radiation is the rate of energy emitted per unit projected area normal to the direction of propagation, per unit solid angle, i.e.

$$I_b = \frac{dQ}{dA_1 \cdot \cos(\theta) \cdot d\omega}$$

i.e.
$$I_b = \frac{dQ}{dA_1 \cdot \cos(\theta) \cdot \frac{dA_2}{r^2}} \quad ...(13.18)$$

But, it is clear from Fig. 13.9 that differential area dA_2 is equal to:

$$dA_2 = (r \cdot d\theta) \cdot (r \cdot \sin(\theta) \cdot d\phi)$$

i.e.
$$dA_2 = r^2 \cdot \sin(\theta) \cdot d\theta \cdot d\phi \quad ...(13.19)$$

Then, from Eqs. 13.18 and 13.19,

$dQ = I_b \cdot dA_1 \cdot \sin(\theta) \cdot \cos(\theta) \cdot d\theta \cdot d\phi$

Then, total rate of radiant energy falling on the hemisphere, Q, is obtained by integrating this value of dQ over the entire hemispherical surface. Noting that the whole of hemispherical surface is covered by taking θ from 0 to $(\pi/2)$ and, ϕ from 0 to $(2.\pi)$, we write:

$$Q = I_b \cdot dA_1 \cdot \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\cdot\pi} \sin(\theta) \cdot \cos(\theta) d\theta\, d\phi$$

i.e. $$Q = 2 \cdot \pi \cdot I_b \cdot dA_1 \cdot \int_{\theta=0}^{\frac{\pi}{2}} \sin(\theta) \cdot \cos(\theta) d\theta$$

i.e. $$Q = \pi \cdot I_b \cdot dA_1 \cdot \int_{\theta=0}^{\frac{\pi}{2}} 2 \cdot \sin(\theta) \cdot \cos(\theta) d\theta$$

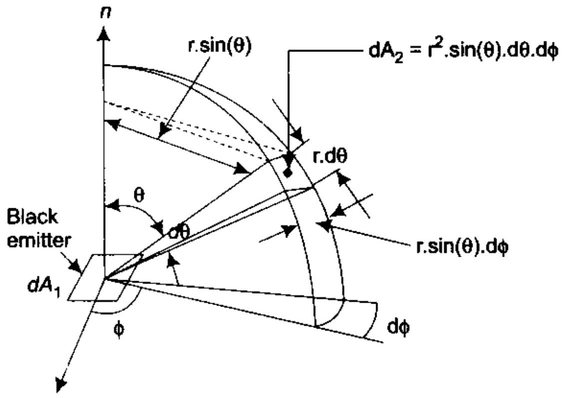


FIGURE 13.9 Radiation from a differential area dA_1 to surrounding hemisphere

i.e. $$Q = \pi \cdot I_b \cdot dA_1 \cdot \int_{\theta=0}^{\frac{\pi}{2}} \sin(2 \cdot \theta) d\theta$$

i.e. $$Q = \pi \cdot I_b \cdot dA_1 \quad ...(13.20)$$

But, Q is also equal to: $E_b \cdot dA_1$

Therefore, $E_b \cdot dA_1 = \pi \cdot I_b \cdot dA_1$

or, $$E_b = \pi \cdot I_b \quad ...(13.21)$$

i.e. **Total emissive power of a black (diffuse) surface is equal to π times the intensity of radiation.**

This is an important relation, which will be used while calculating the view factors required to determine net energy exchange between surfaces.

13.3.6 Emissivity, Real Surface and Grey Surface

As already stated, a 'black body' is an ideal, and it emits maximum amount radiation at a given temperature; a black body also absorbs all the radiation incident on it. A perfect black body does not exist in practice, but this concept is useful as a standard to compare radiation properties of different bodies.

Real surfaces always emit less radiation as compared to a black body.

Emissivity (ε) of a surface is defined as 'the ratio of radiation emitted by a surface to that emitted by a black body at the same temperature'. Value of ε varies between 0 and 1. For a black body, $\varepsilon = 1$, and emissivity of a surface is a measure of how closely that surface approaches a black body.

Emissivity of a surface is not a constant, but depends on nature of the surface, temperature, wavelength, method of fabrication, etc. For example, oxide film on a metal surface increases its emissivity. Emissivity of alloys is greater than that of pure metals. And, emissivity of semi-conductors is greater than 0.8 at 100 deg.C and goes on decreasing with rising temperature. Dielectric materials have higher values of emissivity as compared to that of pure metals, and in this case also, emissivity decreases with increasing temperature.

ε_λ refers to the emissivity at a given wavelength, λ, and is known as **spectral emissivity**. When it is averaged over all wavelengths, it is known as **total emissivity**.

Similarly, ε_θ refers to emissivity in a given direction, θ, where θ is the angle made by the direction considered with the normal to the surface; this is known as **directional emissivity**. When ε_θ is averaged over all directions, it is known as **hemispherical emissivity**. Thus, the **total hemispherical emissivity (ε)** of a surface is the average emissivity over all directions and all wavelengths and is expressed as:

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma \cdot T^4} \quad ...(13.22)$$

where, $E(T)$ is the emissive power of the real surface. Similarly, spectral emissivity is defined as:

$$\varepsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)} \quad ...(13.23)$$

TABLE 13.3 Emissivity values for a few surfaces at room temperature

Surface	ε
Aluminium:	
Polished	0.03
Anodised	0.84
Foil	0.05
Copper:	
Polished	0.03
Tarnished	0.75
Stainless Steel:	
Polished	0.21
Dull	0.60
Concrete	0.88
White marble	0.95
Red brick	0.93
Asphalt	0.90
Black paint	0.97
Snow	0.97
Human skin	0.97

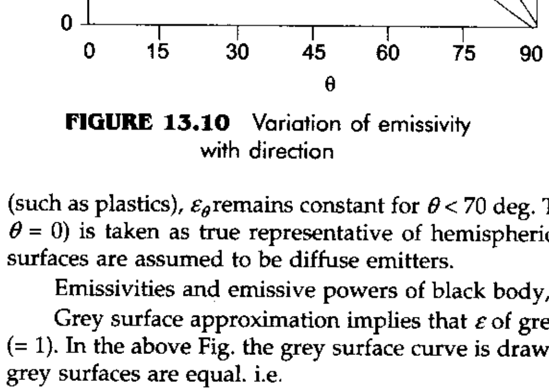


FIGURE 13.10 Variation of emissivity with direction

Emissivity values for a few surfaces at room temperature are given in Table 13.3. More detailed listing is given in Handbooks.

Generally, for simplification of calculations in radiation heat transfer, we make 'grey' and 'diffuse' approximations.

A surface is said to be **grey** if its properties are independent of wavelength, and a surface is **diffuse** if its properties are independent of direction.

A black body is perfectly diffuse, and, real bodies, though not perfectly diffuse, come quite close to it. As an example, a qualitative graph of directional emissivity, ε_θ with θ, for electrical conductors and nonconductors, is given in Fig. 13.10 (θ is measured from the normal to the surface, and $\theta = 0$ means normal to the surface).

It may be observed that for conductors, ε_θ is nearly constant for about $\theta < 40$ deg. and for nonconductors (such as plastics), ε_θ remains constant for $\theta < 70$ deg. Therefore, directional emissivity in the normal direction (i.e. $\theta = 0$) is taken as true representative of hemispherical emissivity; further, in radiation analysis, generally, the surfaces are assumed to be diffuse emitters.

Emissivities and emissive powers of black body, real surface and grey surfaces are compared in Fig. 13.11.

Grey surface approximation implies that ε of grey surface is a constant, but less than that of a black surface (= 1). In the above Fig. the grey surface curve is drawn such that areas under the emission curves of the real and grey surfaces are equal. i.e.

$$\varepsilon(T)\cdot\sigma\cdot T^4 = \int_0^\infty \varepsilon_\lambda(T)\cdot E_{b\lambda}(T)d\lambda$$

Therefore, average emissivity is given by:

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(T)\cdot E_{b\lambda}(T)d\lambda}{\sigma\cdot T^4} \quad ...(13.24)$$

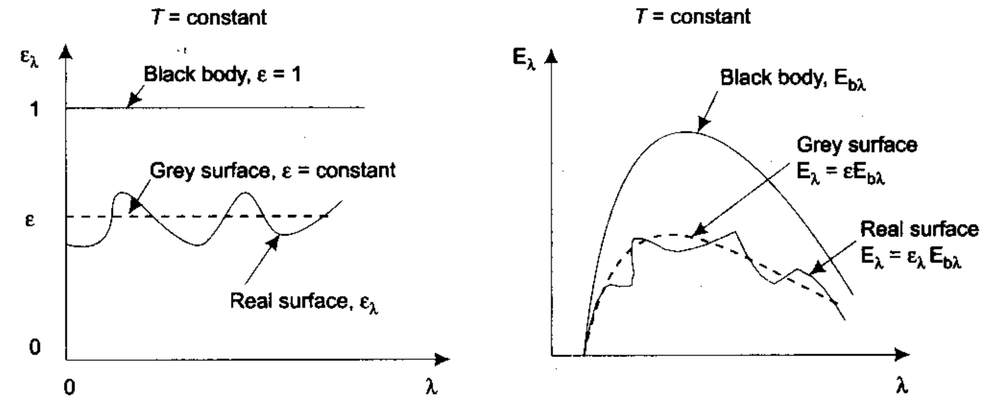


FIGURE 13.11 Emissivity and emissive power for black body, grey and real surfaces at a given temperature

Integrand on the RHS of the above equation has generally to be evaluated numerically. However, if the wavelength spectrum can be divided into sufficient number of wave-bands and the emissivity can be assumed to be constant (but different) in each band, then the integration can be performed quite easily.

For example, let the variation of spectral emissivity with wavelength be as follows:

$$\varepsilon_1 = \text{constant},\ 0 \le \lambda \le \lambda_1$$
$$\varepsilon_2 = \text{constant},\ \lambda_1 \le \lambda \le \lambda_2$$
$$\varepsilon_3 = \text{constant},\ \lambda_2 \le \lambda \le \infty$$

Then, the average emissivity is calculated using Eq. 13.24 as follows:

$$\varepsilon(T) = \frac{\varepsilon_1 \cdot \int_0^{\lambda_1} E_{b\lambda}(T)\,d\lambda}{\sigma \cdot T^4} + \frac{\varepsilon_2 \cdot \int_{\lambda_1}^{\lambda_2} E_{b\lambda}(T)\,d\lambda}{\sigma \cdot T^4} + \frac{\varepsilon_3 \cdot \int_{\lambda_2}^{\infty} E_{b\lambda}(T)\,d\lambda}{\sigma \cdot T^4} \quad \text{...(13.25)}$$

i.e. $$\varepsilon(T) = \varepsilon_1.F_{0-\lambda 1}(T) + \varepsilon_2.F_{\lambda 1-\lambda 2}(T) + \varepsilon_3.F_{\lambda 2-\infty}(T) \quad \text{...(13.26)}$$

Factors $F_{0-\lambda 1}(T)$, etc., can easily be determined using Table 13.2.

It should be clearly understood that emissivity values strongly depend on the surface conditions, oxidation, roughness, cleanliness, type of finish, etc. So, there is always an element of uncertainty while using reported values.

13.3.7 Kirchhoff's Law

Kirchhoff's law establishes a relation between the total, hemispherical emissivity, ε of a surface and the total, hemispherical absorptivity. This is a very useful equation in calculating the net radiant heat loss from surfaces.

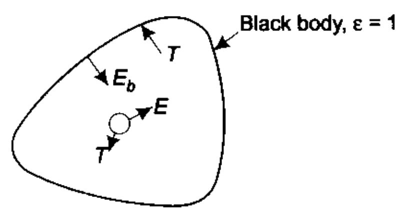


FIGURE 13.12 Kirchhoff's Law

Consider a small, grey body of area A, emissivity ε and temperature T be located inside an isothermal enclosure maintained at the *same temperature* T. Since the enclosure (or, cavity) is isothermal, its behaviour can be taken as that of a black body, irrespective of its surface properties. Also, since the grey body inside the enclosure is small, it does not affect the black body nature of the enclosure.

Now, radiation incident on the small body is equal to the radiation emitted by the black body at temperature T, i.e. $G = E_b(T) = \sigma.T^4$, per unit surface area. And, the radiation absorbed by the small body per unit surface area $= G_{abs} = \alpha.G = \alpha.\sigma.T^4$. Further, radiation emitted by the small body per unit area of its surface $= E = \varepsilon.\sigma.T^4$.

Since both the small body and the enclosure are at the same temperature, T, they will be in thermal equilibrium and the net heat transfer rate to the small body must be equal to zero.

i.e. radiation emitted by the small body = radiation ab-sorbed by the small body,

i.e. $$A.\varepsilon.\sigma.T^4 = A.\alpha.\sigma.T^4$$

or,

$$\varepsilon(T) = \alpha(T) \quad ...(13.27)$$

Eq. (13.27) represents Kirchhoff's law. Kirchhoff's law states that the **"total hemispherical emissivity, ε of a grey surface at a temperature T is equal to its absorptivity, α for black body radiation from a source at the same temperature T."**

Note the important restrictions on Eq. 13.27: one, incident radiation must be from a black body, and, second, black body must be at the same temperature as that of the other body. However, for practical purposes, we assume that the emissivity and absorptivity of a surface are equal, even when that surface is not in thermal equilibrium with the surroundings, since absorptivity of most of the real surfaces is not very much sensitive to temperature and wavelength.

Similar to Eq. 13.27, we can write for monochromatic radiation,

$$\varepsilon_\lambda(T) = \alpha_\lambda(T) \quad ...(13.28)$$

Example 13.1. Incident radiation ($G = 1577\ W/m^2$) strikes an object. The amount of energy absorbed is 472 W/m^2 and the amount of energy transmitted is 78.8 W/m^2. What is the value of reflectivity?

Solution.

Data:

$G := 1577\ W/m^2 \qquad Q_a := 472\ W/m^2 \qquad Q_t := 78.8\ W/m^2$

Let Q_r be the reflected radiation.

Then, we have: $\qquad Q_r := G - Q_a - Q_t$

i.e. $\qquad Q_r = 1.026 \times 10^3\ W/m^2 \qquad$ *(reflected radiation)*

Therefore, reflectivity ρ is given by:

$$\rho := \frac{Q_r}{G}$$

i.e. $\qquad \rho = 0.651 \qquad$ *(reflectivity.)*

Example 13.2. A hole of area $dA = 2\ cm^2$ is opened on the surface of a large spherical cavity whose inside is maintained at 1000 K. Calculate: (a) the radiation energy streaming through the hole in all directions into space, (b) the radiation energy streaming per unit solid angle in a direction making a 60 deg. angle with the normal to the surface of the opening.

Solution. See Fig. Ex. 13.2.

Data:

$dA := 2 \times 10^{-4}\ m^2 \qquad T := 1000\ K \qquad \sigma := 5.67 \times 10^{-8}\ W/(m^2K^4)$ (Stefan–Boltzann const.) $\qquad \theta := 60$ deg.

i.e. $\qquad \theta := 60 \cdot \frac{\pi}{180}$ *(radians)*

(a) Radiation streaming out in all directions:

Since the spherical cavity can be considered as a black body, energy streaming out is given by Stefan–Boltzmann law:

i.e. $\qquad Q := dA \cdot \sigma \cdot T^4$

or, $\qquad Q = 11.34\ W \qquad$ *(radiation energy streaming through the hole.)*

(b) Radiation streaming out through unit solid angle, in a direction making 60 deg. with normal:

Now, we have the relation: $E_b = \pi \cdot I_b$, where E_b = Emissive power, and I_b = Intensity of radiation.

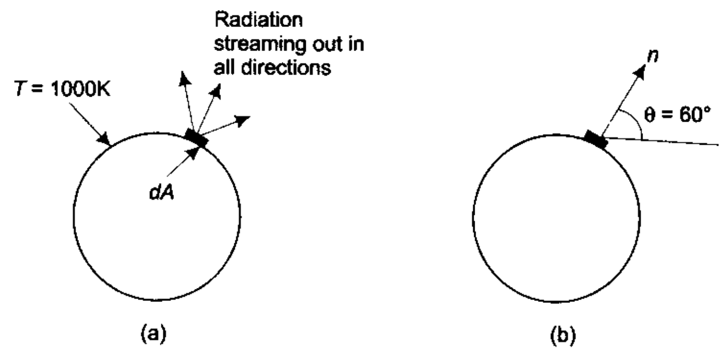


FIGURE Example 13.2 Radiation streaming out from a hole on the surface of a sphere

Therefore, $I_b = \dfrac{E_b}{\pi}$

But, $E_b = \sigma \cdot T^4$ W/m^2 *(for a black body, by Stefan–Boltzmann law)*

i.e. $I_b := \dfrac{\sigma \cdot T^4}{\pi}$ W/(m^{2}sr) *(intensity of radiation)*

i.e. $I_b = 1.805 \times 10^4$ W/(m^{2}sr)

and, radiation streaming out in that direction:

$Q := I_b \cdot dA \cdot \cos(\theta)$ W *(where θ is in radians (Note: while using Mathcad, θ must be in radians while calculating* $\cos(\theta)$.))

i.e. $Q = 1.805$ **W** ***(radiaion through a solid angle of unity, in a direction of 60 deg. with normal to the surface.)***

Example 13.3. It is observed that intensity of radiation is maximum in case of solar radiation at a wavelength of 0.49 microns. Assuming the sun as a black body, estimate its surface temperature and emissive power. Wein displacement constant = 0.289×10^{-2} mK.

Solution.

Data:

$\lambda_{max} := 0.49$ microns $\sigma := 5.67 \times 10^{-8}$ W/(m^{2}K^4) (Stefan–Boltzmann constant)

By Wein's displacement law, we have :

$$\lambda_{max} \cdot T = 2890 \text{ microns K}$$

Therefore,

$$T := \frac{2890}{\lambda_{max}}$$

i.e. $T = 5.898 \times 10^3$ **K** *(surface temperature of sun)*

Heat flux at the surface E_b:

Sun can be considered as a black body; then, from Stefan–Boltzmann law:

$$E_b := \sigma \cdot T^4$$

i.e. $E_b = 6.86 \times 10^7$ W/m^2 *(heat flux at the surface of sun.)*

Example 13.4. The temperature of a body of area 0.1 m^2 is 900 K. Calculate the total rate of energy emission, intensity of normal radiation in W/(m^{2}sr), maximum monochromatic emissive power, and wavelength at which it occurs.

Solution.

Data:

$T := 900$ K $A := 0.1$ m^2 $\sigma := 5.67 \times 10^{-8}$ W/(m^{2}K^4) (Stefan–Boltzmann constant)

Total rate of energy emission:

$E_b := \sigma \cdot T^4$ *(from Stefan–Boltzmann law)*

i.e. $E_b = 3.72 \times 10^4$ W/m^2 *(total emissive power)*

Therefore,

$$Q := E_b \cdot A$$

i.e. $Q = 3.72 \times 10^3$ W *(total energy emission from surface)*

Intensity of normal radiation:

(Note that for a black (diffuse) surface, intensity is the same in all directions.)

$$I := \frac{E_b}{\pi}$$

i.e. $I = 1.184 \times 10^4$ **W/(m^{2}sr)**

Wavelength of maximum monochromatic emissive power:

From Wein's displacement law, we have:

$$\lambda_m \cdot T = 2898 \ \mu\text{mK}$$

Therefore,

$$\lambda_m := \frac{2898}{T} \ \mu\text{m}$$

i.e. $\lambda_m = 3.22$ **μm** ***(wave length for maximum monochromatic emissive power at 900 K.)***

Maximum monochromatic emissive power:

We use Planck's law, with values of constants, C_1 and C_2:

$C_1 := 3.742 \times 10^8$ W/(μm^4/m^2)

and, $C_2 := 1.4387 \times 10^4$ μmK

$$E_{b\lambda max} := \frac{C_1 \cdot \lambda_m^{-5}}{\exp\left(\frac{C_2}{\lambda_m \cdot T}\right) - 1}$$ *(Planck's law)*

i.e. $E_{b\lambda max} := 7.6 \times 10^3$ **W/(m².micron)** ***(maximum monochrome emissive power.)***

Alternatively:

We can directly apply Eq.13.12:

$$E_{b\lambda m} := 1.287 \times 10^{-5} \cdot T^5 \text{ W/m}^3. \quad ...(13.12)$$

i.e. $E_{b\lambda m} = 7.6 \times 10^9 \text{ W/m}^3 = 7.6 \times 10^3$ **W/(m²μm)** ***(same as obtained above.)***

Example 13.5. Window glass transmits radiant energy in the wavelength range 0.4 μm to 2.5 μm. Determine the fraction of total radiant energy which is transmittted, when the source temperature is: (a) 5800 K (i.e. sun's surface temperature), and (b) 300 K (i.e. room temperature).

Solution.

Data:

$T_1 := 5800$ K $\quad T_2 := 300$ K $\quad \lambda_1 := 0.4$ μm $\quad \lambda_2 := 2.5$ μm

$\sigma = 5.67 \times 10^{-8}$ W/(m²K⁴) (Stefan–Boltzmann constant)

Case (a): Source temperature T_1 = 5800 K

We use Table 13.2 where radiation functions are tabulated against the product ($\lambda.T$).

We have:

$$\lambda_1 \cdot T_1 = 2.32 \times 10^3 \ \mu\text{m/K}$$
$$\lambda_2 \cdot T_1 = 1.45 \times 10^4 \ \mu\text{m/K}$$

Corresponding to these λT values, we get, from Table 13.2:

$$F_{0\_\lambda 1} := 0.12454$$ *(by interpolation)*

and, $$F_{0\_\lambda 2} := 0.9661$$

Therefore, fraction transmmitted is equal to:

$$0.9661 - 0.12454 = 0.842$$

i.e. 84.2% of the energy coming from the sun (at 5800 K) is transmitted through the window glass.

Case (b): Source temperature T_2 = 300 K

Again, we use Table 13.2 where, radiation functions are tabulated against the product ($\lambda.T$).

We have:

$$\lambda_1 \cdot T_2 = 120 \ \mu\text{mK}$$
$$\lambda_2 \cdot T_2 = 750 \ \mu\text{mK}$$

Corresponding to these λT values, we get, from Table 13.2:

$$F_{0\_\lambda 2} = 0.0$$

and, $$F_{0\_\lambda 2} = 1.2 \times 10^{-5} = \text{almost zero},$$

i.e. practically no energy will be transmitted through the window glass in this wavelength range, if the source temperature = 300 K. In other words, glass is 'opaque' to radiation at 300 K in the wavelength range 0.4 μm to 2.5 μm.

As mentioned in text, this is the principle of **'Greenhouse effect'**, wherein radiation from a high temperature source (i.e. sun) is allowed to pass through the glass into the enclosure of the greenhouse, while radiation at a relatively low temperature from within the enclosure, is not allowed to escape out. This, in effect, causes an increase in the temperature of the space within the enclosure.

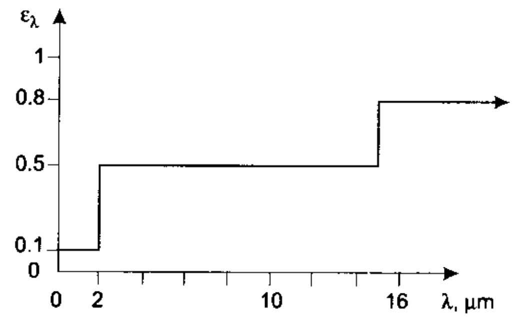


FIGURE Example 13.6 Spectral emissivity distribution against wavelength

Example 13.6. Spectral emissivity of a particular surface at 800 K is approximated by a step function, as follows: $\varepsilon_1 = 0.1$ for λ = 0 to 2 μm, $\varepsilon_2 = 0.5$ for λ = 2 to 15 μm, and $\varepsilon_3 = 0.8$ for λ = 15 to ∞. Calculate (i) the total (hemispherical) emissive power, and (ii) total hemispherical emissivity, ε over all wavelengths.

Solution.

Data:

$T := 800$ K *(temperature)*

$\varepsilon_1 := 0.1$ *(emissivity in wavelength range: 0 to 2 μm)*

$\varepsilon_2 := 0.5$ *(emissivity in wavelength range: 2 to 15 μm)*

$\varepsilon_3 := 0.8$ *(emissivity in wavelength range: 15 μm to ∞.)*

$\sigma := 5.67 \times 10^{-8}$ W/(m²K⁴) *(Stefan–Boltzmann constant)*

$\lambda_1 := 2\ \mu m$
$\lambda_2 := 15\ \mu m$
$\lambda_3 := \infty$

Planck's law gives spectral emissive power of a black body.

For a non-grey surface considered in this problem, we can write:

Total emissive power: $$E = \int_0^\infty \varepsilon_\lambda(\lambda) \cdot E_{b\lambda}\, d\lambda$$

Variation of ε_λ with λ is specified in the problem.

Therefore, splitting the above integral into parts:

$$E = \int_0^2 \varepsilon_\lambda(\lambda) \cdot E_{b\lambda}\, d\lambda + \int_2^{15} \varepsilon_\lambda(\lambda) \cdot E_{b\lambda}\, d\lambda + \int_{15}^\infty \varepsilon_\lambda(\lambda) \cdot E_{b\lambda}\, d\lambda$$

i.e. $$E = 0.1 \cdot \int_0^2 E_{b\lambda}\, d\lambda + 0.5 \cdot \int_2^{15} E_{b\lambda}\, d\lambda + 0.8 \cdot \int_{15}^\infty E_{b\lambda}\, d\lambda$$

Then, $$\varepsilon = \frac{E}{E_b} = \varepsilon_1 \cdot (F_{0\_\lambda 1}) + \varepsilon_2 \cdot (F_{\lambda 1\_\lambda 2}) + \varepsilon_3 \cdot (F_{\lambda 3\_\text{infinity}})$$

i.e. $$\varepsilon = \varepsilon_1 \cdot (F_{0\_\lambda 1}) + \varepsilon_2 \cdot (F_{0\_\lambda 2} - F_{0\_\lambda 1}) + \varepsilon_3 \cdot (F_{0\_\text{infinity}} - F_{\lambda 0\_\lambda 2}) \quad ...(a)$$

Values of $F_{0-\lambda 1}$, etc., are obtained from Table 13.2.

We have

$\lambda_1 \cdot T = 1.6 \times 10^3$ *(correspondingly, we get:* $F_{0\_\lambda 1} = 0.0197$)

$\lambda_2 \cdot T = 1.2 \times 10^4$ *(correspondingly, we get:* $F_{0\_\lambda 2} = 0.9451$)

$\lambda_3 \cdot T = \infty$ *(correspondingly, we get:* $F_{0\_\lambda 3} = 1$)

Then, from Eq. a:

$\varepsilon := 0.0197 + 0.5 \cdot (0.9451 - 0.0197) + 0.8 \cdot (1 - 0.9451)$

i.e. $\varepsilon = 0.516$ ***(Total hemispherical emissivity over all wavelengths.)***

And, total emissive power of this surface is given by:

$E := \varepsilon \cdot \sigma \cdot T^4\ \text{W/m}^2$ *(total emissive power)*

i.e. $E = 1.199 \times 10^4\ \mathbf{W/m^2}$ ***(total emissive power.)***

13.4 The View Factor and Radiation Energy Exchange between Black Bodies

So far, we studied the fundamental laws of radiation and radiative properties of surfaces. But, in practical situations, we are mostly interested in radiative heat exchange between surfaces. The radiative heat exchange may be only between two surfaces, or from one or more surfaces in an enclosure. If the surfaces involved are 'black', then, the problem is simplified since the radiation falling on a black surface is completely absorbed and none is reflected; however, if the surfaces are 'grey', then the problem is slightly more complicated since one has to take into account the multiple reflections from surfaces. In either case, the radiative heat exchange depends on:

(i) absolute temperatures of surfaces
(ii) radiative properties of surfaces, and
(iii) geometry and relative orientation of the surfaces involved.

Point (iii) mentioned above is obvious since, generally, in engineering problems, we assume the surfaces to be 'diffuse', i.e. radiation is emitted in all possible directions, and all of the energy emitted by surface 1 may not be intercepted by surface 2. This statement is quantified by what is known as **'View factor'**. View factor is also known by other names such as: 'configuration factor', 'shape factor', 'angle factor', etc.

View factor is defined as *the fraction of radiant energy leaving one surface which strikes a second surface directly.* Here, 'directly' means that reflection or re-radiated energy is not considered. View factor is denoted by F_{12}, where the first subscript, 1 stands for the emitting surface, and the second subscript, 2 stands for the receiving surface.

We have:

F_{12} = (Direct radiation from surface 1 incident on surface 2) ***divided by*** (Total radiation from emitting surface 1).

We desire to develop a general relation for view factor between two surfaces.

Infinetisimal areas:

As a first step, consider differential areas dA_1 and dA_2 on two black surfaces A_1 and A_2 exchanging heat by radiation only. See Fig. 13.13.

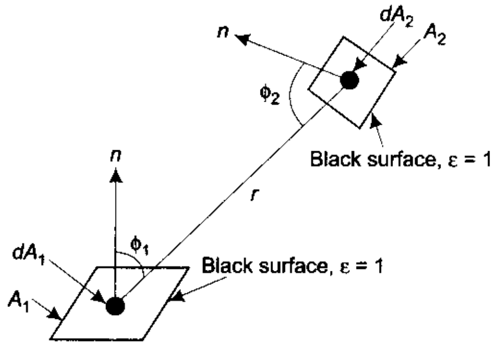


FIGURE 13.13 Areas and angles used in derivation of view factor relation

dA_1 and dA_2 are at a distance 'r' apart and the normals to these areas make angles ϕ_1 and ϕ_2 with the line connecting them, as shown. Then, using the definition of intensity, we can write:

Energy leaving dA_1 and falling on $dA_2 = dQ_{12}$ =
Intensity of black body dA_1 x projected area of dA_1 on a plane perpendicular to line joining dA_1 and $dA2$ x solid angle subtended by dA_2 at dA_1.

i.e. $$dQ_{12} = I_{b1}\cdot(dA_1\cdot\cos(\phi_1))\cdot\left(\frac{dA_2\cdot\cos(\phi_2)}{r^2}\right) \quad ...(13.29)$$

Now, total energy radiated from dA_1 is given by:

$$dQ_1 = E_{b1}\cdot dA_1$$

i.e. $$dQ_1 = (\pi\cdot I_{b1})\cdot dA_1 \quad ...(13.30)$$

Then, by definition, the view factor $F_{dA1\text{-}dA2}$ is the ratio of dQ_{12} to dQ_1:

$$F_{dA1\text{-}dA2} = \frac{\cos(\phi_1)\cdot\cos(\phi_2)\cdot dA_2}{\pi\cdot r^2} \quad ...(13.31)$$

Note that the view factor involves geometrical quantities only.

Eq. 13.31 gives the view factor between two infinetisimal areas. Such a situation is encountered even when finite areas are involved, when the distance between these two areas 'r', is very large.

Infinitesimal to finite area: i.e. the emitter is very small and the receiving surface is of finite size. Here, integration over the entire surface A_2 has to be considered.

Again, remembering the definition of view factor, and forming the ratio of dQ_{12} to dQ_1:

$$F_{dA1-A2} = \frac{\int_{A_2} I_{b1}(\cos(\phi_1)\,dA_1)\cos(\phi_2)\,\frac{dA_2}{r^2}}{\pi I_{b1}\,dA_1}$$

Since both I_{b1} and dA_1 are independent of integration, we can write:

$$F_{dA1-A2} = \int_{A2} \frac{(\cos(\phi_1)\cos(\phi_2)\,dA_2}{\pi r^2} \quad ...(13.32)$$

Practical situation of calculating view factors between infinitesimal to finite areas are encountered in the case of a small thermocouple bead located inside a pipe or a small, spherical point source radiator located by the side of a wall, etc.

Finite to finite area: once again, from the definition of view factor:

$$F_{A1-A2} = \frac{\int_{A1}\int_{A2} I_{b1}(\cos(\phi_1)\,dA_1)\cos(\phi_2)\,\frac{dA_2}{r^2}}{\int_{A1}\pi I_{b1}\,dA_1}$$

For constant I_{b1}, above equation becomes:

$$F_{A1-A2} = \frac{1}{\pi A_1}\int_{A1}\int_{A2}\frac{(\cos(\phi_1)\cos(\phi_2)}{r^2}\,dA_1\,dA_2 \quad ...(13.33)$$

It is clear from Eqs. 13.31, 13.32 and 13.33 that the view factor depends only on the relative orientation (or spatial relation) of the two bodies; it does not depend on the emissivities of the surfaces or the temperatures. Further, also note that the surfaces are assumed to be isothermal and diffuse emitters.

In general, we write Eq. 13.33 compactly as:

$$F_{12} = \frac{1}{\pi A_1}\int_{A1}\int_{A2}\frac{(\cos(\phi_1)\cos(\phi_2)}{r^2}\,dA_1\,dA_2 \quad ...(13.34)$$

Here, F_{12} means 'the view factor from surface 1 to surface 2'.

Similarly, if we desire to get the view factor from surface 2 to surface 1, we simply interchange suffixes 1 and 2:

$$F_{21} = \frac{1}{\pi A_2}\int_{A2}\int_{A1} \frac{(\cos(\phi_2)\cos(\phi_1))}{r^2} dA_2 dA_1 \quad ...(13.35)$$

Note that in Eqs. 13.34 and 13.35, the double integrals differ only in the order of integration, and as such, yield the same result. Then, multiplying Eq. 13.34 by A_1, and Eq. 13.35 by A_2, and equating the double integrals, we get:

$$A_1 \cdot F_{12} = A_2 \cdot F_{21} \quad ...(13.36)$$

Eq. 13.36 is known as **'reciprocity theorem'** and is a very useful and important relation. It helps one to find out one of the view factors when the other one is known. In practice, one of the view factors which is easier to calculate is obtained first, and the other view factor is found out next, by using the reciprocity theorem.

Note: It is easier to remember the view factor relation given in Eq. 13.34 as:

$$A_1 \cdot F_{12} = \int_{A1}\int_{A2} \frac{(\cos(\phi_1)\cos(\phi_2))}{\pi r^2} dA_1 dA_2 . \quad ...(13.37)$$

Radiation energy exchange between black bodies:

As already mentioned, analysis of heat exchange between two black bodies is simpler since a black body absorbs all the radiation impinging on it and none is reflected.

Consider two black surfaces A_1 and A_2 exchanging radiation energy with each other.

Then, rate of energy emitted by surface 1, which directly strikes surface 2 is given by:

$$Q_{12} = A_1 \cdot F_{12} \cdot E_{b1} = A_1 \cdot F_{12} \cdot \sigma \cdot T_1^4 \quad ...(13.38)$$

This energy is completely absorbed by surface 2, since surface 2 is black.

Similarly, of energy emitted by surface 2, which directly strikes surface 1 is given by:

$$Q_{21} = A_2 \cdot F_{21} \cdot E_{b2} = A_2 \cdot F_{21} \cdot \sigma \cdot T_2^4 \quad ...(13.39)$$

and, net radiation exchange between the two surfaces is:

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot T_1^4 - A_2 \cdot F_{21} \cdot \sigma \cdot T_2^4$$

But, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ by reciprocity theorem

Therefore,

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) = A_2 \cdot F_{21} \cdot \sigma \cdot (T_1^4 - T_2^4), \text{ W.} \quad ...(13.40)$$

13.5 Properties of View Factor and View Factor Algebra

We shall enumerate the salient features of view factor. View factors of some geometries are easily calculated; however, more often, calculation of view factors for more complex shapes is quite difficult. In many cases, the complex shapes could be broken down into simpler shapes for whom the view factors are already known or could easily be calculated. Then, with this knowledge, the view factor for the desired complex shape could be calculated by remembering the definition of view factor as the fraction of energy emitted by surface 1, which is intercepted directly by surface 2, and the interrelation between the various view factors. This is known as 'view factor algebra'.

Properties of view factor:

(i) The view factor depends only on the geometrics of bodies involved and not on their temperatures or surface properties.

(ii) Between two surfaces that exchange energy by radiation, the mutual shape factors are governed by the **'reciprocity relation'**, namely, $A_1.F_{12} = A_2.F_{21}$.

(iii) When a convex surface 1 is completely enclosed by another surface 2, it is clear from Fig. 13.14 (a) that all of the radiant energy emitted by surface 1 is intercepted by the enclosing surface 2. Therefore, view factor of surface 1 w.r.t. surface 2 is equal to unity. i.e. $F_{12} = 1$. And, the view factor of surface 2 w.r.t. surface 1 is then easily calculated by applying the reciprocity relation, i.e. $A_1.1 = A_2.F_{21}$, or, $F_{21} = A_1/A_2$.

(iv) Radiation emitted from a flat surface never falls directly on that surface (see Fig. 13.14 (b)), i.e. view factor of a flat surface w.r.t. itself is equal to zero, i.e. $F_{11} = 0$. This is valid for a convex surface too, as shown in Fig. 13.14 (c).

(v) For a concave surface, it is clear from Fig. 13.14 (d) that F_{11} is not equal to zero since some fraction of radiation emitted by a concave surface does fall on that surface directly.

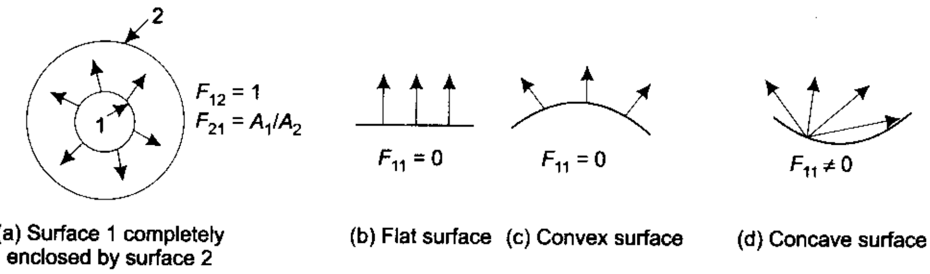


FIGURE 13.14 View factors for a few surfaces

(vi) If two, plane surfaces A_1 and A_2 are parallel to each other and separated by a short distance between them, practically all the radiation issuing from surface 1 falls directly on surface 2, and vice-versa. Therefore, $F_{12} = F_{21} = 1$.

(vii) When the radiating surface 1 is divided into, say, two sub-areas A_3 and A_4, as shown in Fig. 13.15 (a), we have:

$$A_1 \cdot F_{12} = A_3 \cdot F_{32} + A_4 \cdot F_{42} \quad ...(13.41)$$

Obviously, $F_{12} \neq F_{32} + F_{42}$.

(viii) Instead, if the receiving surface A_2 is sub-divided into parts A_3 and A_4 as shown in Fig. 13.15 (b), we have:

$$A_1 \cdot F_{12} = A_1 \cdot F_{13} + A_1 \cdot F_{14} \quad ...(13.42)$$

i.e. $F_{12} = F_{13} + F_{14}$,

i.e. view factor from the emitting surface 1 to a sub-divided receiving surface is simply equal to the sum of the individual shape factors from the surface 1 to the respective parts of the receiving surface. This is known as **'Superposition rule'**.

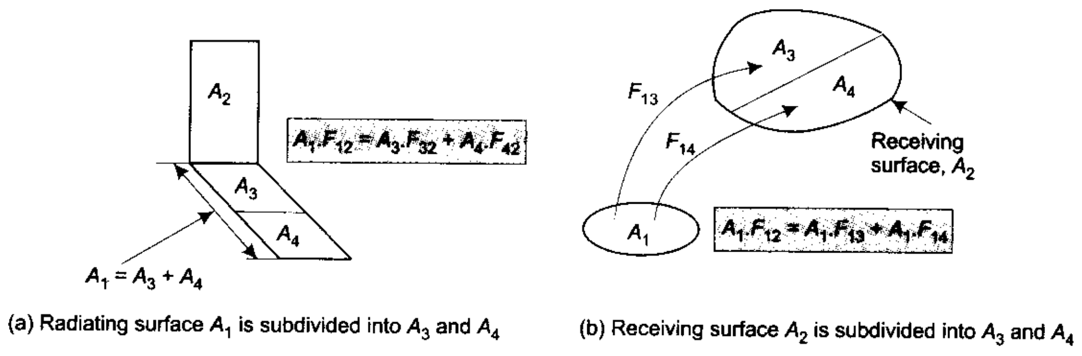


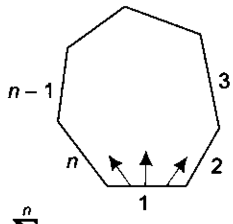
FIGURE 13.15 View factors for sub-divided surfaces

(ix) **Symmetry rule** If two (or more) surfaces are symmetrically located w.r.t. the radiating surface 1, then the view factors from surface 1 to these symmetrically located surfaces are identical. A close inspection of the geometry will reveal if there is any symmetry in a given problem.

(x) **Summation rule** Since radiation energy is emitted from a surface in all directions, invariably, we consider the emitting surface to be part of an enclosure. Even if there is an opening, we consider the opening as a surface with the radiative properties of that opening. Then, the conservation of energy principle requires that sum of all the view factors from the surface 1 to all other surfaces forming the enclosure, must be equal to 1. See Fig. 13.16, where the interior surface of a completely enclosed space is sub-divided into n parts, each of area $A_1, A_2, A_3, ..., A_n$.

Then,

$$F_{11} + F_{12} + ... + F_{1n} = 1$$



$$\sum_{j=1}^{n} F_{ij} = 1 \qquad i = 1, 2, 3, \ldots, n$$

FIGURE 13.16 View factor—summation rule for radiation in an enclosure

$$F_{21} + F_{22} + \ldots + F_{2n} = 1$$

$$F_{n1} + F_{n2} + \ldots + F_{nn} = 1 \qquad \ldots(13.43)$$

i.e.

$$\sum_{j=1}^{n} F_{ij} = 1 \quad i = 1, 2, 3, \ldots n \qquad \ldots(13.44)$$

(xi) In an enclosure of 'n' black surfaces, maintained at temperatures T_1, T_2, ... ,T_n, *net* radiation from any surface, say, surface 1, is given by summing up the net radiation heat transfers from surface 1 to each of the other surfaces of the enclosure:

$$Q_{1_{net}} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) + A_1 \cdot F_{13} \cdot \sigma \cdot (T_1^4 - T_3^4)$$

$$+ A_1 \cdot F_{14} \cdot \sigma \cdot (T_1^4 - T_4^4) + \ldots + A_1 \cdot F_{1n} \cdot \sigma \cdot (T_1^4 - T_n^4) \quad \ldots(13.45)$$

Note: Often, while solving radiation problems, determination of the view factor is the most difficult part. It will be useful to keep in mind the definition of view factor, summation rule, reciprocity relation, superposition rule and symmetry rule while attempting to find out the view factors.

13.6 Methods of Determining View Factors

While solving problems in radiation heat transfer, required numerical values of view factors may be obtained by the following methods:

(i) By performing the necessary integrations in Eqs. 13.31, 13.32 or 13.33. However, except in very simple cases, most of the time, the direct integration procedure is quite difficult.

(ii) Use of readily available analytical formulas or graphs prepared by researchers for the specific geometry in question.

(iii) Use of view factor algebra in conjunction with definition of view factor, summation rule, reciprocity relation, superposition rule and symmetry rule.

(iv) Experimental and graphical techniques.

13.6.1 By Direct Integration

We shall demonstrate direct integration procedure with two examples:

Example 13.7. Find out the view factor from an elemental disk dA_1 to a much larger disk A_2 of radius R, located directly above and parallel to the small disk at a vertical distance L from the small disk, as shown in Fig. Example 13.7.

Solution. Area dA_1 is much smaller than area A_2; so, this is the case of finding out the view factor from a differential area to a finite area. So, we shall apply Eq. 13.32, i.e.

$$F_{dA1-A2} = \int_{A2} \frac{\cos(\phi_1)\cos(\phi_2)dA_2}{\pi r^2} \qquad \ldots(13.32)$$

Now, on area A_2, consider a differential area dA_2 of radius x and width dx as shown. Angles ϕ_1 and ϕ_2, made by the line 'r' connecting dA_1 and dA_2 with the two normals are equal, since the disks are parallel.

We have: $\quad r^2 = x^2 + L^2,\ \phi_1 = \phi_2$

$$\cos(\phi_1) = \frac{L}{r}$$

and, $\quad \cos(\phi_1) = \dfrac{L}{\sqrt{x^2 + L^2}} = \cos(\phi_2)$

and, differential area, $\quad dA_2 = 2 \cdot \pi \cdot x \cdot dx.$

Then, from Eq. 13.32 we get:

$$F_{12} = \int_0^R \frac{2 \cdot L^2 \cdot x}{(x^2 + L^2)^2} dx$$

i.e. $\quad F_{12} = 2 \cdot L^2 \cdot \displaystyle\int_0^R \frac{x}{(x^2 + L^2)^2} dx$

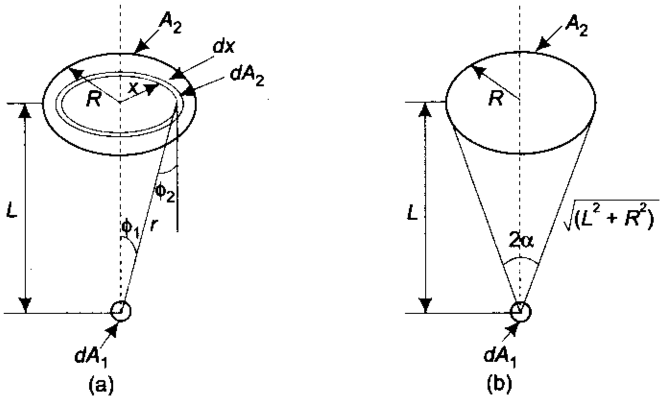


FIGURE Example 13.7 View factor from an elemental area dA_1 to a larger area A_2

Now, let: $u = x^2 + L^2$

Then, $du = 2 \cdot x \cdot dx$

Then, expression for above integral becomes:

$$\int \frac{2 \cdot x}{(x^2 + L^2)^2} dx = \int \frac{1}{u^2} du = \frac{-1}{u} = \frac{-1}{(x^2 + L^2)}$$

Therefore, putting the limits for x from 0 to R:

$$F_{12} = L^2 \cdot \left(\frac{1}{R^2 + L^2} - \frac{1}{L^2} \right)$$

i.e.
$$F_{12} = \frac{R^2}{R^2 + L^2}$$

We can also write:

$$F_{12} = \left(\frac{R}{\sqrt{L^2 + R^2}} \right)^2$$

i.e. $F_{12} = \sin^2(\alpha)$ (*where, 2α is the angle subtended by the area A_2 at dA_1 as shown in Fig. Example* 13.7.)

Example 13.8. Find out the view factors F_{12} and F_{21} between two square surfaces 1 and 2, oriented towards each other as shown in Fig. Example 13.8. Plate 1 has an area of 0.08 m^2 and plate 2 has an area of 0.05 m^2.

Solution.

Since both the plane surfaces are small compared to the distance between them, they can be approximated as differential areas, i.e. we can apply Eq. 13.31 to get the view factors:

$$F_{dA1\_dA2} = \frac{\cos(\phi_1) \cdot \cos(\phi_2) \cdot dA_2}{\pi \cdot r^2} \quad \text{...(13.31)}$$

Data:

$dA_1 := 0.08\ m^2$ $dA_2 := 0.05\ m^2$ $r := 5\ m$ $\phi_1 := 15$ deg. i.e. $\phi_1 := 15 \cdot \frac{\pi}{180}$ radians $\phi_2 := 40$ deg.

i.e. $\phi_2 := 40 \cdot \frac{\pi}{180}$ radians

Note: ϕ_1 and ϕ_2 are expressed in radians, since Mathcad requires that the angles be in radians while evaluating trigonometric functions such as sin (ϕ), cos(ϕ), etc.

Therefore, $\cos(\phi_1) = 0.966$

and, $\cos(\phi_2) = 0.766$

Then, from Eq. 13.31, we get:

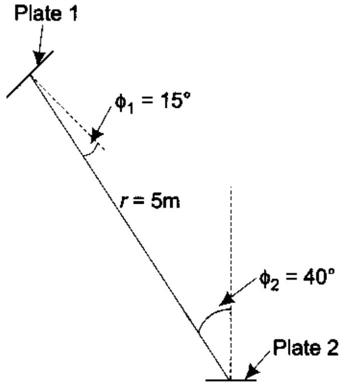


FIGURE Example 13.8 View factors between two small plates separated by a large distance

$$F_{12} = \frac{\cos(\phi_1)\cdot\cos(\phi_2)\cdot dA_2}{\pi\cdot r^2}$$

i.e. $F_{12} = 4.7106 \times 10^{-4}$

i.e. view factor from surface 1 to surface 2 is 0.00047106.

Next, to determine the view factor from surface 2 to 1, we can conveniently use the reciprocity rule, i.e.

$$A_1\cdot F_{12} = A_2\cdot F_{21}$$

In the present case, we write:

$$dA_1\cdot F_{12} = dA_2\cdot F_{21}$$

i.e.
$$F_{21} := \frac{dA_1\cdot F_{12}}{dA_2}$$

i.e.
$$F_{21} = 7.537 \times 10^{-4}$$

i.e. view factor from surface 2 to surface 1 is 0.0007537.

13.6.2 By Analytical Formulas and Graphs

As stated earlier, determination of view factors by direct integration is rather involved because of the contour integrations to be performed over the surfaces. However, several research workers have published analytical relations and graphs for many of the commonly encountered geometries.

Figs. 13.17 and 13.18 show a few two-dimensional and three-dimensional geometries and Tables 13.4 and 13.5 give corresponding view factor relations.

Note: In Table 13.5, atan(x) means arctan(x) or $\tan^{-1}(x)$.

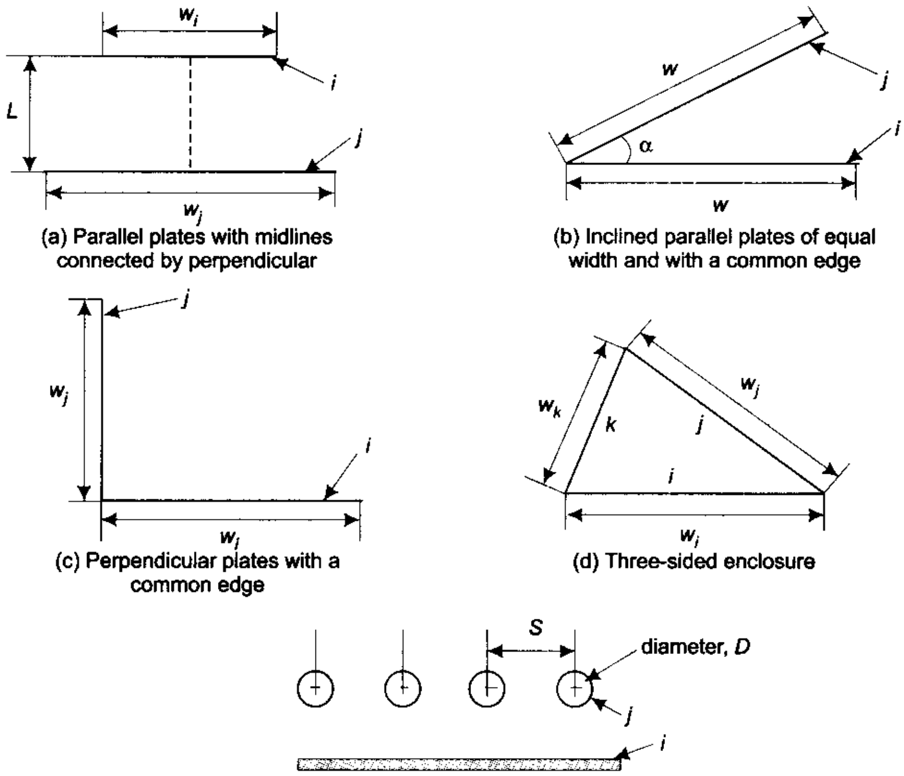
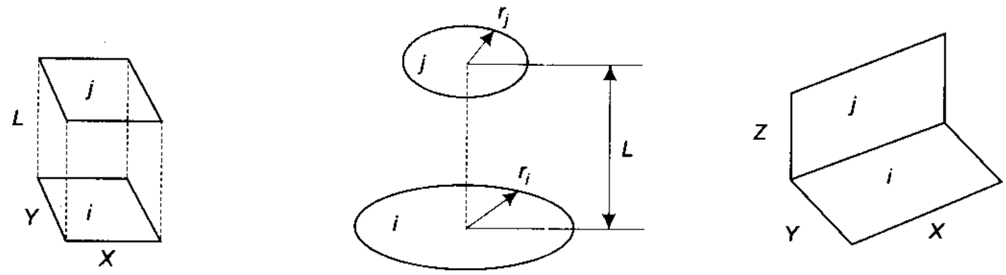


FIGURE 13.17 Few two-dimensional geometries, infinitely long



(a) Aligned parallel rectangles (b) Coaxial parallel disks (c) Perpendicular rectangles with a common edge

FIGURE 13.18 Few three-dimensional geometries

TABLE 13.4 View factors for a few two-dimensional geometries

Geometry	View factor relation
Parallel plates with midlines connected by perpendicular (See Fig. 13.17,a)	$W_i = \frac{w_i}{L} \quad W_j = \frac{w_j}{L}$ $F_{ij} = \frac{\left[(W_i + W_j)^2 + 4\right]^{\frac{1}{2}} - \left[(W_j - W_i)^2 + 4\right]^{\frac{1}{2}}}{2 \cdot W_i}$
Inclined parallel plates of equal width and with a common edge (See Fig. 13.17,b)	$F_{ij} = 1 - \sin\left(\frac{1}{2} \cdot \alpha\right)$
Perpendicular plates with a common edge (See Fig. 13.17,c)	$F_{ij} = \frac{1}{2} \cdot \left[1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{\frac{1}{2}}\right]$
Three-sided enclosure (See Fig. 13.17,d)	$F_{ij} = \frac{w_i + w_j - w_k}{2 \cdot w_i}$
Infinite plane and row of cylinders (See Fig. 13.17,d)	$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{\frac{1}{2}} + \frac{D}{s} \cdot \tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{\frac{1}{2}}$

Table 13.5 gives view factor relations for three important three-dimensional geometries, often required in practice. For example, view factors between aligned parallel rectangles will be useful to calculate heat transfer between the floor and ceiling of a room or a furnace; view factors between coaxial parallel disks will be required to calculate the heat transfer between the top and bottom of a cylindrical furnace, and the view factors between perpendicular rectangles is necessary to calculate the fraction of energy entering a floor through a window on the adjacent wall, or to determine the fraction of energy radiated from the door of a furnace to the floor outside, etc.

It may be observed from the view factor relations given in Table 13.5 that even for these simple cases, the relations are rather complex and difficult to calculate. So, generally, view factors for these (and, many other) geometries are presented in graphical form. It is convenient to use the graphs to determine the view factors quickly, but with the sacrifice of a little accuracy. However, if a computer is available, it is suggested that the analytical relations given in Tables 13.4 and 13.5 could be used for better accuracy.

View factor relation for aligned, parallel rectangles of Fig. 13.18a, is shown in graphical form in Fig. 13.19. This graph is drawn with Mathcad. Here, F_{ij} is plotted against X/L varying from 0.1 to about 30, for given values of Y/L (with Y/L = 0.1, 0.2, 0.4, 0.6, 1.0, 2.0, 4.0 and 10.0).

TABLE 13.5 View factors for a few three-dimensional geometries

Geometry	View factor relation
Aligned parallel rectangles (See Fig. 13.18,a)	$XX = \frac{X}{L} \quad YY = \frac{Y}{L}$ $A = \frac{2}{\pi \cdot XX \cdot YY} \quad B = \ln\left[\sqrt{\frac{(1+XX^2)\cdot(1+YY^2)}{1+XX^2+YY^2}}\right]$ $C = XX\cdot(1+YY^2)^{\frac{1}{2}}\cdot \text{atan}\left[\frac{XX}{(1+YY^2)^{\frac{1}{2}}}\right]$ $D = YY\cdot(1+XX^2)^{\frac{1}{2}}\cdot \text{atan}\left[\frac{YY}{(1+XX^2)^{\frac{1}{2}}}\right]$ $E = XX\cdot \text{atan}(XX) \quad F = YY\cdot \text{atan}(YY)$ $F_{ij} = A\cdot(B + C + D - E - F)$
Coaxial parallel disks (See Fig. 13.17,b)	$R_i = \frac{r_i}{L} \quad R_j = \frac{r_j}{L} \quad S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\cdot\left[S - \left[S^2 - 4\cdot\left(\frac{r_j}{r_i}\right)^2\right]^{\frac{1}{2}}\right]$
Perpendicular rectangles with a common edge (See Fig. 13.18,c)	$H = \frac{Z}{X} \quad W = \frac{Y}{X} \quad A = \frac{1}{\pi\cdot W} \quad B = W\cdot \text{atan}\left(\frac{1}{W}\right)$ $C = H\cdot \text{atan}\left(\frac{1}{H}\right) \quad D = (H^2 + W^2)^{\frac{1}{2}}\cdot \text{atan}\left[\frac{1}{(H^2+W^2)^{\frac{1}{2}}}\right]$ $E = \frac{(1+W^2)\cdot(1+H^2)}{(1+W^2+H^2)}\cdot\left[\frac{W^2\cdot(1+W^2+H^2)}{(1+W^2)\cdot(W^2+H^2)}\right]^{W^2}\cdot\left[\frac{H^2\cdot(1+H^2+W^2)}{(1+H^2)\cdot(H^2+W^2)}\right]^{H^2}$ $F_{ij} = A\cdot\left(B + C - D + \frac{1}{4}\cdot\ln(E)\right)$

Graph of view factor for coaxial parallel disks (of Fig. 13.18,b) is drawn using Mathcad and is shown in Fig. 13.20. Here, view factor F_{ij} is plotted against L/r_i for different values of r_j/L.

And, graph of view factors for perpendicular rectangles with a common edge (of Fig. 13.18,c), drawn using Mathcad, is shown in Fig. 13.21. Here, view factor F_{ij} is plotted against Z/X for different values of Y/X.

Another practically important geometry is that of two concentric cylinders of *finite* length. View factors associated with this geometry are shown in Fig. 13.22.

We shall illustrate the use of analytical relations for view factors given in Table 13.5 or the Figs. 13.19 to 13.21, with an example:

Example 13.9. Find out the net heat transferred between two circular disks 1 and 2, oriented one above the other, parallel to each other on the same centre line, as shown in Fig. Example 13.9. Disk 1 has a radius of 0.5 m and is maintained at 1000 K, and disk 2 has a radius of 0.6 m and is maintained at 600 K. Assume both the disks to be black surfaces.

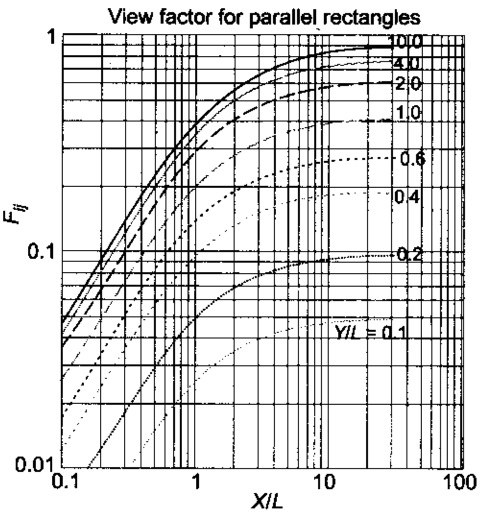


FIGURE 13.19 View factor for aligned, parallel rectangles (See Fig. 13.18a)

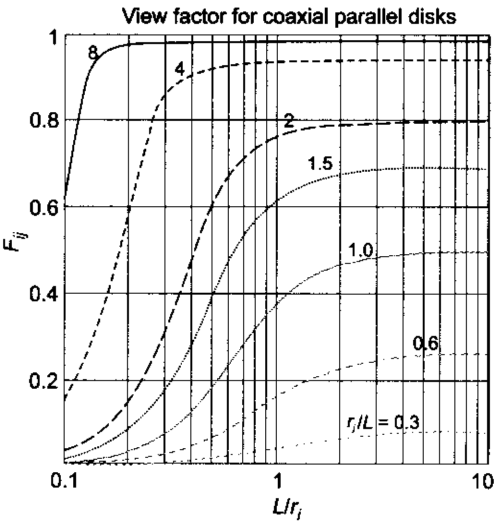


FIGURE 13.20 View factor for coaxial, parallel disks (See Fig. 13.18b)

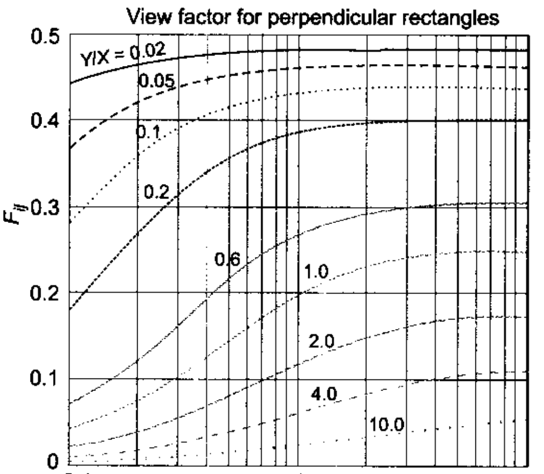


FIGURE 13.21 View factors for coaxial, perpendicular rectangles with a common edge (See Fig. 13.18c)

Solution.

Data:

$r_i := 0.5$ m $\quad r_j := 0.6$ m $\quad L := 1$ m $\quad T_1 := 1000$ K $\quad T_2 := 600$ K $\quad \sigma := 5.67 \times 10^{-8}$ W/m^{2}K

$A_1 := \pi \cdot r_i^2$ i.e. $A_1 = 0.785$ m^2 $\quad A_2 := \pi \cdot r_j^2$ i.e. $A_2 = 1.131$ m^2

This is the case of heat transfer between two black surfaces. So, we use Eq. (13.40), i.e.

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4)$$
$$= A_2 \cdot F_{21} \cdot \sigma \cdot (T_1^4 - T_2^4), \text{ W} \quad \text{...(13.40)}$$

So, the problem reduces to calculating the view factor F_{12} or F_{21}. We can easily find out F_{12} using Fig. 13.20. However, we can determine F_{12} analytically more accurately with Mathcad using the view factor relation given in Table 13.5 for coaxial parallel disks.

We re-write the view factor relation given in Table 13.5 as follows, for ease of calculation with Mathcad:

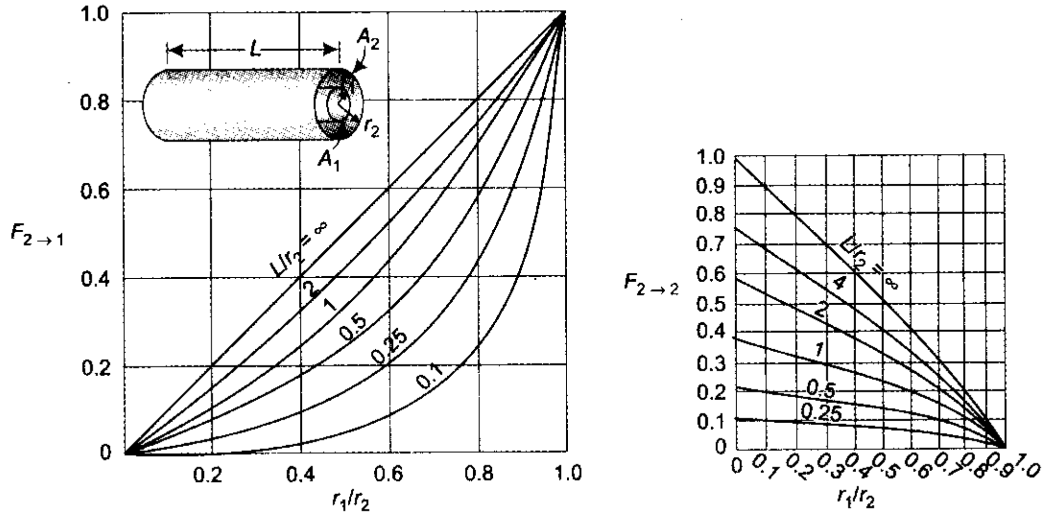


FIGURE 13.22 View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder (b) outer cylinder to itself (Source: Cengel, Yunus A. [1998]. *Heat Transfer: A Practical Approach*. Pub.: McGraw-Hill)

$$R_i := \frac{r_i}{L} \quad R_j = \frac{r_j}{L} \quad S(R_i, R_j) = 1 + \frac{1+R_j^2}{R_i^2}$$

$$F_{12'}(R_i \cdot R_j) := \frac{1}{2}\cdot\left[S(R_i, R_j) - \left[S(R_i, R_j)^2 - 4\cdot\left(\frac{R_j}{R_i}\right)^2\right]^{\frac{1}{2}}\right]$$

(*view factor for coaxial parallel disks*)

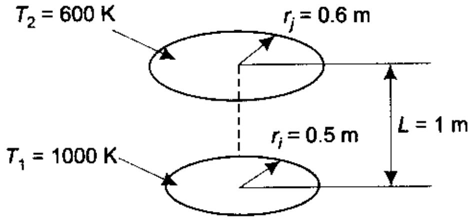


FIGURE Example 13.9 Coaxial parallel disks

Here, first, S is written as a function of R_i and R_j where, $R_i = r_i/L$ and $R_j = r_j/L$. Then, F_{12} is expressed as a function of R_i and R_j. Now, F_{12} is easily obtained for *any* values of R_i and R_j by simply writing $F_{12}\cdot(R_i, R_j) =$.

Therefore, in this case, $R_i = 0.5$

and, $R_j = 0.6$

We get: $F_{12}(0.5, 0.6) = 0.232$

Verify This result may be verified from Fig. 13.20 where, F_{12} is plotted against L/r_i for various values of r_j/L. Now, for our problem, $L/r_i = 1/0.5 = 2$, and $r_j/L = 0.6/1 = 0.6$. Then, from Fig. 13.20, we read $F_{12} = 0.232$, approximately,

i.e. $F_{12} := 0.232$

Therefore, net transfer between disks 1 and 2:

$$Q_{net} := A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) \text{ W} \qquad \textit{(from Eq. 13.40)}$$

i.e. $Q_{net} = 8.992 \times 10^3$ **W.**

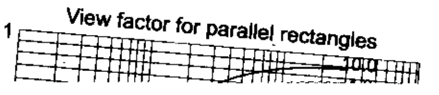
13.6.3 By Use of View Factor Algebra

Often, we have to find out view factors for geometries for which readily no analytical relations or graphs are available. In such cases, sometimes, it may be possible to get the required view factor in terms of view factors of already known geometries, by suitable manipulation using view factor algebra. For this purpose, we remember the definition of view factor (as the fraction of energy emitted by surface 1 and directly falling on surface 2), and invoke the summation rule, reciprocity relation, and inspection of geometry.

We shall illustrate this procedure with some important examples:

Example 13.10. Find out the net heat transferred between the areas A_1 and A_2 shown in Fig. Example 13.10. Area 1 is maintained at 700 K, and area 2 is maintained at 400 K . Assume both the surfaces to be black.

Solution. This is the case of heat transfer between two black surfaces. So, we use Eq. 13.40 i.e.



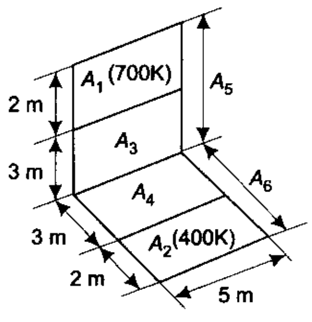


FIGURE Example 13.10 Perpendicular rectangles with a common edge

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) = A_2 \cdot F_{21} \cdot \sigma \cdot (T_1^4 - T_2^4),\ \text{W} \qquad ...(13.40)$$

So, the problem reduces to calculating the view factor F_{12} or F_{21}. We see that to calculate F_{12} for areas A_1 and A_2 as oriented in the Fig. Example 13.10 we do not readily have an analytical relation or a graph. Let us denote the combined areas $(A_1 + A_3)$ by A_5 and $(A_2 + A_4)$ by A_6. Then, we see that A_5 and A_6 are perpendicular rectangles which have a common edge, and we have graphs or analytical relation for the view factor for such an orientation. Then, we resort to view factor algebra, as follows:

Remember that by definition, view factor F_{12} is the fraction of radiant energy emitted by surface 1 which falls directly on surface 2. Looking at the Fig. Example 13.10 we can say that fraction of energy leaving A_1 and falling on A_2 is equal to the fraction falling on A_6 minus the fraction falling on A_4.

i.e. $\quad F_{12} = F_{16} - F_{14}$ *(by definition of view factor)*

i.e. $\quad F_{12} = F_{61} \cdot \dfrac{A_6}{A_1} - F_{41} \cdot \dfrac{A_4}{A_1}$ *(since by reciprocity relation, $A_1 \cdot F_{16} = A_6 \cdot F_{16}$, and $A_1 \cdot F_{14} = A_4 \cdot F_{41}$.)*

i.e. $\quad F_{12} = \dfrac{A_6}{A_1} \cdot (F_{65} - F_{63}) - \dfrac{A_4}{A_1} \cdot (F_{45} - F_{43})$ *(Eq. A ... using the definition of view factor, as done in first step above)*

Now, observe that view factors F_{65}, F_{63}, F_{45} and F_{43} refer to perpendicular rectangles with a common edge, and can be readily obtained from Fig. 13.21, or by analytical relation given in Table 13.5.

We re-write the view factor relation for perpendicular rectangles with a common edge, given in Table 13.5 as follows, for ease of calculation with Mathcad:

$$H := \frac{Z}{X} \quad W := \frac{Y}{X} \quad A(W) := \frac{1}{\pi \cdot W} \quad B(W) = W \cdot \text{atan}\left(\frac{1}{W}\right)$$

$$C(H) := H \cdot \text{atan}\left(\frac{1}{H}\right) \quad D(H, W) := (H^2 + W^2)^{\frac{1}{2}} \cdot \text{atan}\left[\frac{1}{(H^2 + W^2)^{\frac{1}{2}}}\right]$$

$$E(H, W) := \frac{(1+W^2)\cdot(1+H^2)}{(1+W^2+H^2)} \cdot \left[\frac{W^2 \cdot (1+W^2+H^2)}{(1+W^2)\cdot(W^2+H^2)}\right]^{W^2} \cdot \left[\frac{H^2 \cdot (1+H^2+W^2)}{(1+H^2)\cdot(H^2+W^2)}\right]^{H^2}$$

$$F_{ij}(H, W) := A(W) \cdot \left(B(W) + C(H) - D(H,W) + \frac{1}{4} \cdot \ln(E(H,W))\right)$$

(Eq. B...view factor for coaxial perpendicular rectangles with a common edge)

To find F_{65}:

$X := 5 \quad Y := 5 \quad Z := 5$ *(w.r.t. Fig. 13.18 (c) and Fig. Example 13.12)*

$H := \dfrac{Z}{X}$ i.e. $H = 1$

$W := \dfrac{Y}{X}$ i.e. $W = 1$

Therefore,

$F_{ij}(1, 1) = 0.2$ *(substituting in Eq. B)*

i.e. $\quad F_{65} := 0.2$ *(view factor from area A_6 to A_5)*

Note: This value can be verified from Fig. 13.21 also.

To find F_{63}:

$X := 5 \quad Y := 5 \quad Z = 3$ *(w.r.t. Fig. 13.18 (c) and Fig. Example 13.12)*

$H := \dfrac{Z}{X}$ i.e. $H = 0.6$

$W := \dfrac{Y}{X}$ i.e. $W = 1$